National
Qualifications
2022

2022 Mathematics

Paper 1 – (Non-calculator)

National 5

Finalised Marking Instructions

General marking principles for National 5 Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

generic scheme — this indicates why each mark is awarded
illustrative scheme — this covers methods which are commonly seen throughout the marking

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each ○. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example 6 x 6 = 12, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

	$x^2 + 5x + 7 = 9x + 4$
This is a transcription error and so the mark is not awarded.	$x - 4x + 3 = 0$
This is no longer a solution of a quadratic equation, so the mark is not awarded.	$x = 1$

The following example is an exception to the above

	$x^2 + 5x + 7 = 9x + 4$
This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.	$x - 4x + 3 = 0$
	$(x-3)(x-1) = 0$
	$x = 1$ or 3

(i) **Horizontal/vertical marking**
If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bigcirc^5$	$\bigcirc^6$
$\bigcirc^5$	$x = 2$	$x = -4$
$\bigcirc^6$	$y = 5$	$y = -7$

Horizontal: $\bigcirc^5$ $x = 2$ and $x = -4$ $\quad$ Vertical: $\bigcirc^5$ $x = 2$ and $y = 5$
$\bigcirc^6$ $y = 5$ and $y = -7$ $\quad$ $\bigcirc^6$ $x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\qquad$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0\cdot 3}$ must be simplified to 50 $\qquad$ $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $(x^3+2x^2+3x+2)(2x+1)$ written as

  $(x^3+2x^2+3x+2)\times 2x+1$

  $=2x^4+5x^3+8x^2+7x+2$

  gains full credit

- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Marking Instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.			**Method 1** •[1] start calculation correctly •[2] consistent answer in simplest form **Method 2** •[1] start calculation correctly •[2] consistent answer in simplest form	**Method 1** •[1] $\frac{4}{20}+\frac{15}{20}$ •[2] $\frac{19}{30}$ **Method 2** •[1] $\frac{2}{15}+\frac{6}{12}$ or $\frac{2}{15}+\frac{1}{2}$ •[2] $\frac{19}{30}$	2

Notes:

1. Correct answer with no working award 0/2

2. Final answer must be in simplest form

   eg for $\frac{38}{60}$ — award 1/2 ✓✓2

3. •[2] is only available where simplifying is required

4. For subsequent incorrect working, •[2] is not available

   eg for $\frac{19}{30}=1\frac{11}{30}$ — award 1/2 ✓✗

Commonly Observed Responses:

1. For an answer of $\frac{8}{27}$ obtained from

   (a) Method 1: $\frac{2}{3}\left(\frac{1}{5}+\frac{3}{4}\right)=\frac{2}{3}\times\frac{4}{9}=\frac{8}{27}$ — award 0/2

   (b) Method 2: $\frac{2}{3}\left(\frac{1}{5}+\frac{3}{4}\right)=\frac{2}{15}+\frac{6}{12}=\frac{8}{27}$ — award 1/2 ✓

Question			Generic scheme	Illustrative scheme	Max mark
2.			•[1] substitute into x^3-2 •[2] evaluate	•[1] $(-3)^3-2$ •[2] -29	**2**

Notes:

1. Correct answer without working — award 2/2
2. Accept -3^3-2 for •[1]
3. For subsequent incorrect working •[2] is not available eg see COR 3(b)

Commonly Observed Responses:

1. $(-3)^2-2=7$ — award 0/2 ✗✓2
2. (a) $(-3)^3-2=25$ — award 1/2 ✓✗
   (b) $3^3-2=25$ — award 0/2 ✗✓2
3. (a) $-3=(-3)^3-2 \rightarrow -3=-29$ — award 2/2
   (b) $-3=(-3)^3-2 \rightarrow -3=-29 \rightarrow x=-26$ — award 1/2 ✓✗

Question			Generic scheme	Illustrative scheme	Max mark
3.			•[1] correct substitution into formula for volume of cone •[2] calculate volume (calculation must involve a product of at least four numbers including a fraction and 3.14)	•[1] $\frac{1}{3}\times3.14\times10^2\times60$ •[2] 6280 (cm^3)	**2**

Notes:

1. Correct answer without working — award 0/2

Commonly Observed Responses:

1. $\frac{1}{3}\times3.14\times20^2\times60=25\,120$ — award 1/2 ✗✓1
2. $\frac{1}{3}\times3.14\times20\times60=1256$ — award 1/2 ✗✓1
3. $\frac{1}{3}\times3.14\times10\times60=628$ — award 1/2 ✗✓1

Question			Generic scheme	Illustrative scheme	Max mark
4.			•[1] calculate size of angle COE or EDO or OED •[2] calculate size of angle OCE •[3] calculate size of angle ACE	•[1] COE = 112 or EDO = 56 or OED = 56 •[2] OCE = 34 •[3] ACE = 124	**3**

Notes:

1. •[1] and •[2] may be awarded for information marked on the diagram.
2. Where information is not marked on the diagram then working must clearly attach calculations to named angles.
3. For the award of •[3] the answer of 124 must be stated outwith the diagram **or** ACE clearly indicated with an arc and 124.
4. For an answer of 124 with no relevant working award 0/3
5. Degrees signs are not required

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
5.	(a)		•[1] correct bracket with square •[2] complete process consistently	•[1] $(x+4)^2 \ldots$ •[2] $(x+4)^2 - 1$	2

Notes:

1. Correct answer without working — award 2/2
2. Answer for •[2] must be consistent with •[1]
   - eg $(x-4)^2 - 1$ — award 1/2 ✗✓1
   - $(x \pm 8)^2 - 49$ — award 1/2 ✗✓1
   - $(x \pm 8)^2 - 1$ — award 0/2

Commonly Observed Responses:

No working necessary

1. Award 2/2 for
   - (a) $(x+4)^2 + -1$ or $(x+4)^2 + (-1)$
   - (b) $(x+4)(x+4) - 1$
2. Award 1/2 ✗✓1 for
   - (a) $(x \pm 4) - 1$
   - (b) $(x^2 \pm 4) - 1$
   - (c) $(x^2 \pm 4)^2 - 1$
   - (d) $(x \pm 4x)^2 - 1$
   - (e) $(x^2 \pm 4x)^2 - 1$

	(b)		•[3] state coordinates of turning point	•[3] $(-4, -1)$	1

Notes:

1. Answer must be consistent with (a) unless candidate uses method in note 2
2. Accept correct answer obtained by factorising, finding roots and using symmetry
3. Accept $x = -4$, $y = -1$
4. •[3] is not available where brackets are omitted, unless answer is in the form shown in note 3

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
6.			**Method 1:** $y-b=m(x-a)$		3
			•1 calculate gradient	•1 -4 or equivalent	
			•2 substitute gradient and a point into $y-b=m(x-a)$	•2 eg $y-7=-4(x-(-5))$	
			•3 determine the equation of the line in simplest form	•3 $y=-4x-13$ or equivalent	
			Method 2: $y=mx+c$		
			•1 calculate gradient	•1 -4 or equivalent	
			•2 substitute gradient and a point into $y=mx+c$	•2 eg $7=-4\times(-5)+c$	
			•3 determine the equation of the line in simplest form	•3 $y=-4x-13$ or equivalent	

Notes:

1. Correct answer without working award 0/3

2. (a) Accept $-\frac{8}{2}$ for the award of •1

   (b) BEWARE •1 is not available for $\frac{7-(-1)}{-5-(-3)}=\frac{-8}{2}=-\frac{8}{2}$ or $\frac{(-1)-7}{-3-(-5)}=\frac{8}{-2}=-\frac{8}{2}$

3. For an incorrect simplification of a gradient, a mark is not awarded at the point where the error occurs eg

   (a) $-\frac{8}{2}=4\rightarrow\quad 7=4\times(-5)+c\rightarrow\quad y=4x+27$ award 2/3 ×✓1✓1

   (b) $-\frac{8}{2}\rightarrow\quad 7=4\times(-5)+c\rightarrow\quad y=4x+27$ award 2/3 ✓×✓1

   (c) $-\frac{8}{2}\rightarrow\quad 7=-\frac{8}{2}\times(-5)+c\rightarrow\quad y=4x+27$ award 2/3 ✓✓×

Commonly Observed Responses:

Working must be shown.

1. $y=-\frac{4}{1}x-13$ award 2/3 ✓✓×

Question			Generic scheme	Illustrative scheme	Max mark
7.			•¹ multiply by C^2 •² subtract 4	•¹ $C^2D = B + 4$ •² $B = C^2D - 4$ or equivalent	**2**

Notes:

1. Correct answer without working — award 0/2

2. BEWARE $D = \frac{B+4}{C^2} \rightarrow D - 4 = \frac{B}{C^2} \rightarrow C^2D - 4 = B$ — award 0/2

3. For subsequent incorrect working, •² is not available

Commonly Observed Responses:

1. $C^2 \times D = B + 4 \rightarrow B = C^2 \times D - 4$ — award 2/2

2. $D = \frac{B+4}{C^2} \rightarrow D - 4 = \frac{B}{C^2} \rightarrow B = C^2(D-4)$ — award 1/2 ✓1✗

2. $\sqrt{C} \times D = B + 4 \rightarrow B = \sqrt{C} \times D - 4$ — award 1/2 ✗✓1

Question			Generic scheme	Illustrative scheme	Max mark
8.	(a)		•¹ state the value of a	•¹ 3	**1**

Notes:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•² state the value of b	•² 8	**1**

Notes:

1. For $(y=)3\sin 8x$ — award 1/1 for (a) and 1/1 for (b)
2. For answers of $a = 8$ and $b = 3$ or $(y=)8\sin 3x$ — award 0/1 ✗ for (a) and 1/1✓1 for (b)

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
9.			•¹ correct substitution into cosine rule •² calculate $\cos B$ in simplest form	•¹ $(\cos B =)\dfrac{3^2+7^2-5^2}{2\times3\times7}$ •² $\dfrac{11}{14}$	**2**

Notes:

1. Correct answer without working award 0/2
2. Accept $5^2 = 3^2 + 7^2 - 2\times3\times7\times\cos B$ for •¹
3. •² is only available where simplifying is required

Commonly Observed Responses:

1. $\dfrac{3^2+7^2-5^2}{2\times3\times7} \to \dfrac{33}{42}$ award 1/2 ✓✓2
2. $\dfrac{3^2+5^2-7^2}{2\times3\times5} \to -\dfrac{1}{2}$ award 1/2 ✗✓1
3. $\dfrac{5^2+7^2-3^2}{2\times5\times7} \to \dfrac{13}{14}$ award 1/2 ✗✓1

Question			Generic scheme	Illustrative scheme	Max mark
10.			•[1] know that $70\% = £16.10$ •[2] begin valid strategy •[3] complete calculation within valid strategy	•[1] $70\% = £16.10$ •[2] $(10\%=)\dfrac{16.10}{7}$ or $(1\%=)\dfrac{16.10}{70}$ or equivalent •[3] $(£)23$	3

Notes:

1. Correct answer without working — award 0/3

2. (a) $70\% = £16.10 \rightarrow 30\%$ of $16.10 = 4.83$ — award 1/3 ✓××
   (b) 30% of $16.10 = 4.83$ award 0/3

3. (a) $70\% = £16.10 \rightarrow 70\%$ of $16.10 = 11.27$ — award 1/3 ✓××
   (b) 70% of $16.10 = 11.27$ award 0/3

4. (a) $70\% = £16.10 \rightarrow 130\%$ of $16.10 = 20.93$ — award 1/3 ✓××
   (b) 130% of $16.10 = 20.93$ award 0/3

Commonly Observed Responses:

1. $\dfrac{16.1}{0.7} = 23$ — award 3/3

2. (a) $30\% = 16.10 \rightarrow \dfrac{16.1}{0.3} = 53.66$ or 53.67 — award 2/3 ×✓1✓1

   (c) $\dfrac{16.1}{0.3} = 53.66$ or 53.67 — award 1/3 ××✓1

3. (a) $130\% = 16.10 \rightarrow \dfrac{16.1}{1.3} = 12.38$ — award 2/3 ×✓1✓1

   (b) $\dfrac{16.1}{1.3} = 12.38$ — award 1/3 ××✓1

Question			Generic scheme	Illustrative scheme	Max mark
11.			**Method 1**	**Method 1**	3
			•¹ apply $(m^a)^b = m^{ab}$	•¹ m^{-8}	
			•² apply $m^a \times m^b = m^{a+b}$	•² m^{-13}	
			•³ apply $m^{-a} = \dfrac{1}{m^a}$	•³ $\dfrac{1}{m^{13}}$	
			Method 2	**Method 2**	
			•¹ apply $(m^a)^b = m^{ab}$	•¹ m^{-8}	
			•² apply $m^{-a} = \dfrac{1}{m^a}$	•² $\dfrac{1}{m^8}$ or $\dfrac{1}{m^5}$	
			•³ complete simplification	•³ $\dfrac{1}{m^{13}}$	
			Method 3	**Method 3**	
			•¹ apply $m^{-a} = \dfrac{1}{m^a}$	•¹ $\left(\dfrac{1}{m^2}\right)^4$ or $\dfrac{1}{m^5}$	
			•² apply $\left(\dfrac{1}{m^a}\right)^b = \dfrac{1}{m^{ab}}$	•² $\dfrac{1}{m^8}$	
			•³ complete simplification	•³ $\dfrac{1}{m^{13}}$	

Notes:

1. Correct answer without working award 3/3

Commonly Observed Responses:

1. $m^2 \times m^{-5} \to \dfrac{1}{m^3}$ award 2/3 ✗✓1✓1
2. $m^8 \times m^{-5} \to m^3$ award 1/3 ✗✓1✗

Question			Generic scheme	Illustrative scheme	Max mark
12.			•¹ start to divide fractions •² simplify	•¹ $\ldots\times\frac{(x+2)^2}{5}$ •² $\frac{4(x+2)}{5}$ or $\frac{4x+8}{5}$	2

Notes:

1. Correct answer without working — award 0/2

2. Accept $\frac{4}{5}(x+2)$ for the award of •²

3. •¹ is available for eg $\frac{4(x+2)^2}{(x+2)(x+2)^2} \div \frac{5(x+2)}{(x+2)^2(x+2)} \rightarrow \frac{4(x+2)^2}{(x+2)(x+2)^2} \times \frac{(x+2)^2(x+2)}{5(x+2)}$

4. For subsequent incorrect working, •² is not available

   eg $\frac{4(x+2)}{5} = \frac{4x+2}{5}$

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
13.			•¹ expand bracket •² express surd in simplest form •³ complete simplification	•¹ $\sqrt{100}-\sqrt{20}\ldots$ •² $\ldots2\sqrt{5}\ldots$ •³ $10+6\sqrt{5}$	3

Notes:

1. Correct answer without working — award 0/3

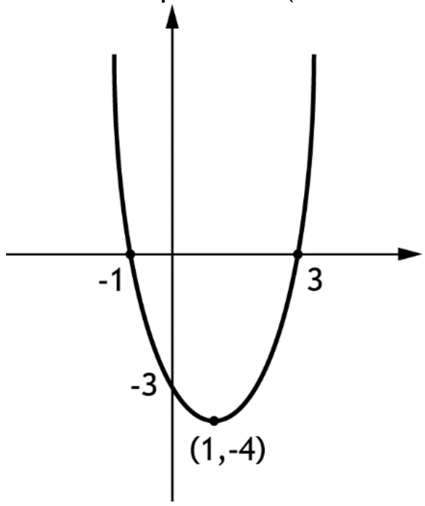
2. For the award of •¹ accept eg
   (a) $\sqrt{10}\times\sqrt{10}-\sqrt{10}\times\sqrt{2}$
   (b) $\sqrt{5}\sqrt{2}\sqrt{5}\sqrt{2}-\sqrt{5}\sqrt{2}\sqrt{2}$

2. •³ is **not** available for:
   (a) a collection of terms which simplify to a single term
   eg $\sqrt{80}-\sqrt{20}+8\sqrt{5} \rightarrow 4\sqrt{5}-2\sqrt{5}+8\sqrt{5} \rightarrow 10\sqrt{5}$ — award 1/3 ✗✓✓2
   (b) A collection of terms with only one surd term
   eg $\sqrt{100}-\sqrt{20}+8\sqrt{5} \rightarrow 50-10+8\sqrt{5} \rightarrow 40+8\sqrt{5}$ — award 1/3 ✓✗✓2

4. For subsequent incorrect working, •³ is not available

Commonly Observed Responses:

1. $\sqrt{10}\left(\sqrt{10}-\sqrt{2}\right)+8\sqrt{5} \rightarrow \sqrt{10}\left(\sqrt{8}\right)+8\sqrt{5} \rightarrow 4\sqrt{5}+8\sqrt{5} \rightarrow 12\sqrt{5}$ — award 1/3 ✗✓1✓2

Question			Generic scheme	Illustrative scheme	Max mark
14.			•[1] identify roots •[2] identify turning point **OR** y-intercept •[3] identify turning point **AND** y-intercept and sketch a consistently annotated parabola	•[1] -1 **AND** 3 •[2] $(1,-4)$ **OR** -3 •[3] $(1,-4)$ **AND** -3 and a consistently annotated parabola (see note 2) 	3

Notes:

1. •[1]and •[2] may be awarded for roots, and turning point or y -intercept indicated on the graph (no additional working required)

2. •[3] is only available where the roots, turning point **AND** y-intercept are clearly marked and consistently annotated on the sketch

3. Accept correctly calculated roots and/or y-intercept marked as $(0,-1)$, $(0, 3)$ **and** $(-3, 0)$ as evidence for the award of •[3] (treat as bad form)

4. •[3] is not available if the graph is not a parabola

   eg roots -3 and 1 $\rightarrow$ turning point $(-1, 0)$ or y-intercept -3 award 1/3 ✗✓1✗

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
15.	(a)		•[1] construct expression for area of triangle	•[1] $\frac{3}{2}(x+12)$	1

Notes:

1. Accept eg $\frac{1}{2}\times 3\times(x+12)$, $\frac{1}{2}3(x+12)$, $3(x+12)\div 2$, $1.5(x+12)$, $\frac{3(x+12)}{2}$

2. For $\frac{1}{2}\times 3\times x+12$
   (a) accept as bad form if **correct** expansion appears in part (b)
   (b) do not accept otherwise

3. Do not penalise **subsequent** incorrect expansion of bracket in part (a)
   eg (a) $\frac{3}{2}(x+12)=3x+18$ award 1/1
   (b) $3x+18$ award 0/1

4. If no expression appears in part (a), accept answer to part (a) written in part (b)

Commonly Observed Responses:

1. $\frac{3}{2}(x+12)\sin \mathrm{C}$ award 0/1

Question			Generic scheme	Illustrative scheme	Max mark
15	(b)		•² construct expression for area of rectangle and equate to area of triangle	•² $\frac{3}{2}(x+12)=6(8-x)$	4
			•³ start to solve equation	•³ $3(x+12)=12(8-x)$ or $\frac{3}{2}x+18=6(8-x)$	
			•⁴ re-arrange equation	•⁴ $15x = 60$ or $7.5x = 30$ or equivalent	
			•⁵ solve for x	•⁵ $x=4$	

Notes:

1. For guess and check — award 0/4

2. •³ is not available if the expression for the area of the triangle does not include a fraction

   eg for an answer of $3(x+12)$ in part (a):

   $3(x+12)=6(8-x)\rightarrow 9x=12\rightarrow x=\frac{4}{3}$ — award 3/4 ✓1×✓1✓1

3. Do not award •⁵ for a decimal approximation to a fraction.
   However, do not penalise incorrect conversion to a mixed number or decimal approximation following a fraction answer (in its simplest form)

   (a) $3(x+12)=6(8-x)\rightarrow 9x=12\rightarrow x=1.\dot{3}$ — award 3/4 ✓1×✓1✓1

   (b) $3(x+12)=6(8-x)\rightarrow 9x=12\rightarrow x=\frac{4}{3}\rightarrow x=1.33\ldots$ — award 3/4 ✓1×✓1✓1

   (c) $3(x+12)=6(8-x)\rightarrow 9x=12\rightarrow x=1.33\ldots$ — award 2/4 ✓1×✓1✓2

4. If solution to part (a) contains $\sin C$, only •² and •³ are available:

   eg $\frac{3}{2}(x+12)\sin C=6(8-x)\rightarrow 3(x+12)\sin C=12(8-x)$ — award 2/4 ✓1✓1××

5. •⁵ is not available for division by a single digit leading to an integer answer

   eg (a) $\ldots\rightarrow 9x=12\rightarrow x=\frac{4}{3}$ — award •⁵

   (b) $\ldots\rightarrow 6x=48\rightarrow x=8$ — do **not** award •⁵

Commonly Observed Responses:

[END OF MARKING INSTRUCTIONS]