National Qualifications 2025

2025 Mathematics of Mechanics

Advanced Higher

Question Paper Finalised Marking Instructions

These marking instructions have been prepared by examination teams for use by SQA appointed markers when marking external course assessments.

General marking principles for Advanced Higher Mathematics of Mechanics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

The marking instructions for each question are generally in two sections:

generic scheme — this indicates why each mark is awarded
illustrative scheme — this covers methods which are commonly seen throughout the marking

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each ○. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example $6 \times 6 = 12$, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.

This is no longer a solution of a quadratic equation, so the mark is not awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$x = 1$$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$(x-3)(x-1) = 0$$
$$x = 1 \text{ or } 3$$

(i) Horizontal/vertical marking

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bigcirc^5$	$\bigcirc^6$
$\bigcirc^5$	$x = 2$	$x = -4$
$\bigcirc^6$	$y = 5$	$y = -7$

Horizontal: $\bigcirc^5$ $x = 2$ and $x = -4$ Vertical: $\bigcirc^5$ $x = 2$ and $y = 5$
$\bigcirc^6$ $y = 5$ and $y = -7$ $\bigcirc^6$ $x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0{\cdot}3}$ must be simplified to 50 $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 144 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $\left(x^3 + 2x^2 + 3x + 2\right)(2x + 1)$ written as

  $\left(x^3 + 2x^2 + 3x + 2\right) \times 2x + 1$

  $= 2x^4 + 5x^3 + 8x^2 + 7x + 2$

  gains full credit

- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Note: Marking from Image (MFI) annotation change from 2025

A double cross-tick is used to indicate correct working which is irrelevant or insufficient to score any marks. In MFI marking instructions prior to 2025 this was shown as $\ddot{u}_2$ or ü2.

From 2025, the double cross-tick will no longer be used in MFI. A cross or omission symbol will be used instead.

Marking Instructions for each question

Question	Generic scheme	Illustrative scheme	Max mark
1.	•[1] substitute values into equation of motion •[2] calculate time of flight •[3] calculate range	•[1] $0 = (20\sin 33°)t - \frac{1}{2}gt^2$ •[2] 2.22 seconds •[3] 37.3 metres	**3**
Alternative Method 1			
	•[1] state range formula •[2] substitute values into formula •[3] calculate range	•[1] $R = \frac{U^2 \sin 2\theta}{g}$ •[2] $R = \frac{20^2 \sin(2\times 33°)}{9.8}$ •[3] 37.3 metres	**3**
Alternative Method 2			
	•[1] substitute values into equation of motion to find time to max height •[2] calculate time of flight •[3] calculate range	•[1] $0 = 20\sin 33° - gt$ •[2] 2.22 seconds •[3] 37.3 metres	**3**
Notes: 1. Accept 37m.			
Commonly Observed Responses: **COR 1** $t = \frac{-20\sin 33 - 20\sin 33}{-g}$ **or** $\frac{-20\sin 33}{-g}\times 2$			

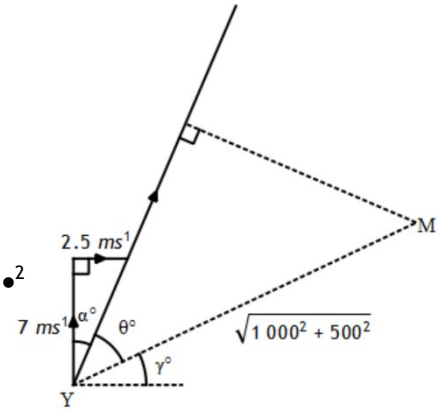
Question			Generic scheme	Illustrative scheme	Max mark
2.			•[1] evidence use of product rule with one term correct •[2] complete differentiation	•[1] $4e^{4x}\sec 3x + e^{4x}\ldots$ **OR** $e^{4x}\sec 3x\tan 3x\times 3 + \sec 3x\ldots$ •[2] $4e^{4x}\sec 3x + 3\mathrm{e}^{4x}\sec 3x\tan 3x$	**2**
Notes: 1. Where •[2] has been awarded, disregard any further working subsequent to the correct answer.					
Commonly Observed Responses: Accept use of quotient rule $\dfrac{4e^{4x}\cos 3x - \ldots}{\cos^2 3x}$ or $\dfrac{\ldots - e^{4x}\times -\sin 3x\times 3}{\cos^2 3x}$ $= \dfrac{4e^{4x}\cos 3x + 3e^{4x}\sin 3x}{\cos^2 3x}$					
3.			•[1] use connection between power, force and velocity •[2] use Newton's 2nd Law •[3] calculate acceleration	•[1] $18000 = Fv$ or $F = \dfrac{18000}{v}$ or $F = \dfrac{18000}{5}$ •[2] $\dfrac{18000}{v} - 450 = 1500a$ •[3] $2.1\,\mathrm{ms}^{-2}$	**3**
Notes:					
Commonly Observed Responses: Energy cannot be used. Evidence of $F = 1500a + 450$ can gain •[2].					

Question			Generic scheme	Illustrative scheme	Max mark
4.			$\bullet^1$ differentiate $\bullet^2$ rewrite integral $\bullet^3$ complete integration	$\bullet^1$ $\frac{du}{dx} = 5\cos 5x$ $\bullet^2$ $\int u^3 . \frac{1}{5}\, du$ $\bullet^3$ $\frac{1}{20}\sin^4 5x + c$	**3**

Notes:

1. Disregard the omission of the constant of integration.
2. Where the integrand contains terms in x, $\bullet^2$ is still available provided these terms are clearly and correctly 'cancelled out'.
3. Where candidates attempt to integrate an expression containing both u and x, where x is either inside the integrand or taken outside as a constant, $\bullet^2$ and $\bullet^3$ are not available.
4. Accept $\frac{1}{20}(\sin 5x)^4 + c$
5. Do not accept $\frac{1}{4}\left(\frac{1}{5}\ldots\right)$

Commonly Observed Responses:

Question	Generic scheme	Illustrative scheme	Max mark
5.	•[1] determine the position of the motorboat or yacht at time t	•[1] $\begin{pmatrix}1000\\500-7t\end{pmatrix}$ or $\begin{pmatrix}2.5t\\0\end{pmatrix}$	**5**
	•[2] determine the relative position of the motorboat and the yacht	•[2] $\begin{pmatrix}1000-2.5t\\500-7t\end{pmatrix}$	
	•[3] determine an expression for the magnitude squared	•[3] $(1000-2.5t)^2+(500-7t)^2$	
	•[4] differentiate and equate to 0	•[4] $110.5t-12000=0$ **or** equivalent (108.59...)	
	•[5] determine the shortest distance	•[5] 774 metres	
Notes: 1. Accept 770 metres.			
Alternative Method 1			
	•[1] construct triangle for relative velocity with yacht or motorboat brought to rest	•[1] eg	
	•[2] construct triangle for relative position of the motorboat and the yacht	•[2] 	
	•[3] determine angles of the relative velocity and relative position	•[3] e.g $\alpha=19.65...$ $\gamma=26.56...$ $\theta=43.78...$	
	•[4] determine initial distance between motorboat and yacht	•[4] 1118.03...m = $500\sqrt{5}$ m	
	•[5] use right-angled triangle to find closest distance	•[5] 774m	
Notes: 1. •[4] can only be awarded if there is clear evidence of relative position in the candidate's solution.			

Question	Generic scheme	Illustrative scheme	Max mark
Alternative Method 2			
	$\bullet^1$ determine the relative velocity	$\bullet^1$ $\begin{pmatrix}-2.5\\-7\end{pmatrix}$	**5**
	$\bullet^2$ determine the relative position of the motorboat and the yacht	$\bullet^2$ $\begin{pmatrix}1000-2.5t\\500-7t\end{pmatrix}$	
	$\bullet^3$ evaluate the scalar product	$\bullet^3$ $2.5^2t-2500+49t-3500$	
	$\bullet^4$ equate to 0 and solve for t	$\bullet^4$ $t=108.59...$	
	$\bullet^5$ determine the shortest distance	$\bullet^5$ 774 metres	
Notes:			
Commonly Observed Responses:			

Question			Generic scheme	Illustrative scheme	Max mark
6.			$\bullet^1$ find $\frac{dy}{dt}$ and $\frac{dx}{dt}$ $\bullet^2$ find $\frac{dy}{dx}$ and equate to 1 or state $\frac{dy}{dt}=\frac{dx}{dt}$ $\bullet^3$ solve for t and calculate position	$\bullet^1$ $2t$ and 4 $\bullet^2$ $\frac{2t}{4}=1$ or $2t=4$ $\bullet^3$ $\begin{pmatrix}11\\9\end{pmatrix}$ metres	**3**

Notes:

1. Accept (11,9) for $\bullet^3$.
2. Only accept $4=2t$ if there is clear evidence of $\frac{dy}{dt}=\frac{dx}{dt}$.

Commonly Observed Responses:

COR 1

Candidate uses $x=y$ leading to $t^2-4t+2=0$, $t=2+\sqrt{2}$ or $2-\sqrt{2}$

Only mark $\bullet^3$ can be awarded, for (16.66, 16.66) **and/or** (5.34, 5.34).

COR 2

Candidate finds $\frac{dy}{dt}$ and $\frac{dx}{dt}$, but then goes on to solve $x=y$.

Award $\bullet^1$. Consider $x=y$ as second strategy and mark as seen.

Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)		$\bullet^1$ integrate acceleration to find speed and consider constant of integration $\bullet^2$ equate expression for speed to $24\,\text{ms}^{-1}$ and solve for t $\bullet^3$ integrate speed to find displacement $\bullet^4$ substitute for t and evaluate	$\bullet^1$ $v = 1.5t^2 + c, \quad c = 0$ $\bullet^2$ 4 seconds $\bullet^3$ $s = 0.5t^3 + d, \quad d = 0$ $\bullet^4$ 32 metres	**4**

Notes:

1. At $\bullet^3$, do not penalise a repeated lack of a constant of integration, or the usage of the same letter a second time.
2. For a candidate who includes the first constant, but omits the second, do not award $\bullet^3$, but $\bullet^4$ is still available.

Commonly Observed Responses:

COR 1

Candidate uses SUVAT leading to e.g. $s = \frac{3t^3}{2}$, $v = 3t^2, t = \sqrt{8}, s = 33.9\,\text{m}$.

Award a maximum of one mark, $\bullet^2$ for $t = \sqrt{8} = \sqrt[4]{64} = 2.83$ seconds.

	(b)		$\bullet^5$ state valid explanation	$\bullet^5$ eg the bike reaching its maximum speed as total resistive forces will balance the traction force.	**1**

Notes:

1. Comments must refer to at least one of:
   - power/force limits time
   - rider limits time, because acceleration reaches >4g after 13s.
   - bike limits time, with reference to engine.
   - terminal velocity reached as friction and air resistance will increase i.e. maximum speed will be reached
2. Where the answer contains incorrect (rather than insufficient) information (before, between or after correct information) $\bullet^5$ is not available.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
8.	(a)		•[1] resolve forces parallel to the plane •[2] resolve forces perpendicular to the plane •[3] combine equations and substitute values •[4] calculate coefficient of friction	•[1] $F = \mu R + mg\sin\theta$ •[2] $R = mg\cos\theta$ •[3] $100 = \mu \times 12g\cos 35° + 12g\sin 35°$ •[4] 0.338	**4**
Notes: 1. Accept 0.34, but not 0.3					
Commonly Observed Responses: **COR 1** Candidate resolves horizontally and vertically. •[1] $R\cos\theta + 100\sin\theta - mg - \mu R\sin\theta = 0$ •[2] $100\cos\theta - R\sin\theta - \mu R\cos\theta = 0$ •[3] $\mu = \dfrac{100\cos^2\theta + 100\sin^2\theta - mg\sin\theta}{mg\cos\theta}$ or equivalent •[4] 0.338 Award full marks.					
	(b)		•[5] resolve forces parallel to the plane •[6] substitute and calculate magnitude of force	•[5] $F + \mu R = mg\sin\theta$ •[6] 34.9 N	**2**
Notes: 1. Accept answers which round to 35 N eg 34.7 for candidates who use 0.34 from part (a).					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
9.			$\bullet^1$ form integral with limits	$\bullet^1$ $\pi\int_{-3}^{3} y^2\,dx$	**3**
			$\bullet^2$ square y and integrate	$\bullet^2$ $\pi\left[\frac{x^5}{5}+\frac{4x^3}{3}+4x\right]_{-3}^{3}$	
			$\bullet^3$ substitute to find volume	$\bullet^3$ 607 (cubic units)	

Notes:

1. For award of $\bullet^1$
   a. Limits must appear at some point
   b. dx must appear at some point
2. Accept $\frac{966}{5}\pi$ for $\bullet^3$.
3. $\bullet^3$ is unavailable for a negative volume or working which leads to zero.

Commonly Observed Responses:

COR 1

Wrong strategy for integration eg $\int (x^2+2)^2\,dx = \frac{(x^2+2)^3}{6x}$, do not award marks $\bullet^2$ and $\bullet^3$.

COR 2

If integration is eased then mark $\bullet^2$ is unavailable (GMP d) but $\bullet^3$ may be awarded.

COR 3

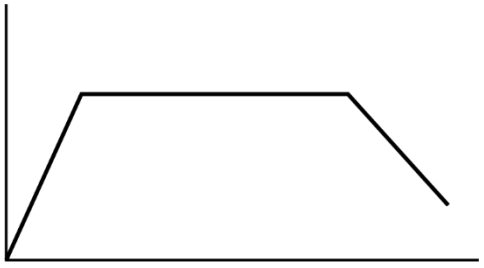
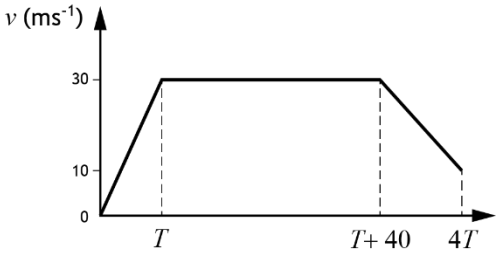
If $y^2 = x^4 + 4$ is used, volume is 381 $=121.2\pi$ (cubic units) for which $\bullet^3$ can be awarded.

Question			Generic scheme	Illustrative scheme	Max mark
10.	(a)	(i)	$\bullet^1$ calculate speed directly across the river $\bullet^2$ calculate crossing time	$\bullet^1$ 4 ms^{-1} $\bullet^2$ 25 seconds	**2**
Notes:					
Commonly Observed Responses: **COR 1** Speed = $\sqrt{5^2+3^2}=\sqrt{34}$; $t=17.15$ seconds do not award $\bullet^1$ and $\bullet^2$. **COR 2** Speed = 5 ; $t=20$ seconds do not award $\bullet^1$ and $\bullet^2$. Exception: If there is clear evidence that the speed directly across the river is either $\sqrt{34}$ or 5 then $\bullet^2$ is available. Note the evidence (i.e. diagram) for this may be in (ii).					
		(ii)	$\bullet^3$ calculate angle to the bank	$\bullet^3$ 53.1°	**1**
Notes: 1. $\bullet^3$ can only be awarded if the angle to the bank upstream is calculated. Accept 126.9°. 2. $\bullet^3$ can be awarded for 59.0° from incorrect setup in (i). 3. $\bullet^3$ is unavailable for subsequent incorrect working.					
Commonly Observed Responses:					
	(b)		$\bullet^4$ calculate new crossing time $\bullet^5$ calculate distance	$\bullet^4$ 20 seconds $\bullet^5$ 60 metres	**2**
Alternative Method					
			$\bullet^4$ find ratio of speeds $\bullet^5$ calculate distance	$\bullet^4$ $(\tan\alpha=)\frac{3}{5}$ $\bullet^5$ 60 metres	
Notes: 1. If angle found and rounded, distance must round to 60 metres for the award of $\bullet^5$. 2. If the correct setup appears in part (b) but this has already been used in part (a): if $\bullet^2$ has been awarded, then $\bullet^4$ is unavailable, but $\bullet^5$ is available; if no marks have been awarded in part (a), then $\bullet^4$ and $\bullet^5$ are available. 3. Correct answer without working award 0/2.					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
11.	(a)		•[1] use Hooke's Law in equilibrium •[2] substitute and calculate modulus of elasticity	•[1] $\frac{\lambda x}{l} = mg$ •[2] 49 N	**2**
Notes: 1. Ignore units of extension and natural length, as long as they are consistent.					
Commonly Observed Responses:					
	(b)		•[3] calculate tension in the spring •[4] write equilibrium equation •[5] substitute normal reaction and calculate coefficient	•[3] 11.76 N or $\frac{49 \times 0.048}{0.2}$ •[4] $11.76 - \mu R = 0$ or $\frac{\lambda x}{l} = \mu R$ •[5] 0.8	**3**
Notes:					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
12.			$\bullet^1$ calculate change in velocity $\bullet^2$ use work energy principle $\bullet^3$ calculate distance	$\bullet^1$ 4 ms^{-1} $\bullet^2$ $\frac{1}{2}mu^2 = \mu Rs$ $\bullet^3$ 2.72 metres	**3**
AlternativeMe Method1					
			$\bullet^1$ calculate change in velocity $\bullet^2$ calculate acceleration $\bullet^3$ calculate distance	$\bullet^1$ 4 ms^{-1} $\bullet^2$ -2.94 ms^{-2} $\bullet^3$ 2.72 metres	**3**
Alternative Method 2					
			$\bullet^1$ calculate change in velocity $\bullet^2$ calculate time using $I = Ft$ $\bullet^3$ calculate distance	$\bullet^1$ 4 ms^{-1} $\bullet^2$ $\frac{40}{3g}$ seconds $\bullet^3$ 2.72 metres	**3**
Notes: 1. $\bullet^3$ can be awarded for a distance calculated from a spurious acceleration.					
Commonly Observed Responses: **COR 1** Candidate uses $v = 0$, leads to $u = -4$. Do not award $\bullet^1$, but $\bullet^2$ and $\bullet^3$ are still available.					

Question			Generic scheme	Illustrative scheme	Max mark
13.			$\bullet^1$ state auxiliary equation	$\bullet^1$ $m^2+m-20=0$	**5**
			$\bullet^2$ state general solution	$\bullet^2$ $y=Ae^{4x}+Be^{-5x}$	
			$\bullet^3$ differentiate	$\bullet^3$ $\frac{dy}{dx}=4Ae^{4x}-5Be^{-5x}$	
			$\bullet^4$ form equations and solve for one constant	$\bullet^4$ $A=3$ or $B=-2$	
			$\bullet^5$ solve for second constant and state particular solution	$\bullet^5$ $y=3e^{4x}-2e^{-5x}$	
Notes: 1. $\bullet^1$ is not available where $''=0''$ has been omitted. 2. Do not award $\bullet^5$ if $''y=''$ does not appear at that stage.					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
14.	(a)		$\bullet^1$ sketch of graph	$\bullet^1$ 	**2**
			$\bullet^2$ all 5 annotations correct	$\bullet^2$ 	

Notes:

1. If graph drawn to zero at $4T$, do not award $\bullet^1$ and $\bullet^2$, however $\bullet^3$, $\bullet^4$, $\bullet^5$ are still available.
2. Ignore solid straight line down at $4T$.
3. Ignore anything after $4T$.

Commonly Observed Responses:

	(b)		$\bullet^3$ start to set up initial equation, with two out of three areas	$\bullet^3$ e.g. $\frac{1}{2}\times T\times 30+40\times 30+\ldots\left(=2500\right)$	**3**
			$\bullet^4$ complete set up	$\bullet^4$ $\ldots+10\left(4T-(T+40)\right)+\frac{1}{2}\left(4T-(T+40)\right)\times 20=2500$	
			$\bullet^5$ solve	$\bullet^5$ 112 seconds	

Notes:

1. For $\bullet^5$, the expression at $\bullet^4$ must contain at least two terms with T.

Commonly Observed Responses:

COR 1

Candidate uses triangle instead of full trapezium, having drawn the correct graph:

$\bullet^4$ is unavailable but $\bullet^3$ and $\bullet^5$ are available (151 seconds).

Question			Generic scheme	Illustrative scheme	Max mark
15.			$\bullet^1$ integrate to find $"uv-"$ $\bullet^2$ differentiate to find $"\int u'v\,dx"$ $\bullet^3$ start to integrate a second time $\bullet^4$ obtain full solution	$\bullet^1$ $9x^2.\frac{1}{3}e^{3x}-\ldots$ $\bullet^2$ $\ldots\int 18x.\frac{1}{3}e^{3x}\,dx$ $\bullet^3$ $\ldots\left(6x.\frac{1}{3}e^{3x}-\int 6.\frac{1}{3}e^{3x}\,dx\right)$ $\bullet^4$ $3x^2e^{3x}-2xe^{3x}+\frac{2}{3}e^{3x}+c$	**4**

Notes:

1. Disregard the omission of the constant of integration.
2. Disregard the omission of dx at $\bullet^2$ and $\bullet^3$.

Commonly Observed Responses:

COR 1

Candidate chooses to differentiate e^{3x} and integrate $9x^2$. Do not award $\bullet^1$ but $\bullet^2$ and $\bullet^3$ are available. $\bullet^4$ is unavailable as the integration cannot be completed.

COR 2

Candidate communicates that $v'=e^{3x}, v=3e^{3x}$. Do not award $\bullet^1$ but $\bullet^2$, $\bullet^3$ and $\bullet^4$ are available.

Leads to $27x^2e^{3x}-162xe^{3x}+486e^{3x}+c$

COR 3 Tabular method

	D	I
+	$9x^2$	e^{3x}
-	$18x$	$\frac{1}{3}e^{3x}$
+	18	$\frac{1}{9}e^{3x}$
-	0	$\frac{1}{27}e^{3x}$

$\bullet^1$ first three rows (including headings – although these may vary).

$\bullet^2$ final two rows.

$\bullet^3$ $9x^2.\frac{1}{3}e^{3x}-18x.\frac{1}{9}e^{3x}+\ldots$

$\bullet^4$ $3x^2e^{3x}-2xe^{3x}+\frac{2}{3}e^{3x}+c$

Question			Generic scheme	Illustrative scheme	Max mark
16.	(a)	(i)	•[1] resolve forces vertically •[2] resolve horizontally •[3] calculate angle	•[1] $T\cos\theta = mg$ or 0.784 •[2] $T\sin\theta = \frac{mv^2}{r}$ or 3.136 •[3] 76.0°	**3**
Notes: 1. Accept 76° or 1.3 radians.					
Commonly Observed Responses: **COR 1** Candidate resolves incorrectly: do not award •[1], however •[2] and •[3] are still available. •[1] $T\sin\theta = mg$ or 0.784 •[2] $T\cos\theta = \frac{mv^2}{r}$ or 3.136 •[3] 14°					
		(ii)	•[4] calculate tension	•[4] 3.23 N	**1**
Notes: 1. Ac16cept 3.2N or 3.24N					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
16.	(b)	(i)	$\bullet^5$ find expression for vertical distance and prove	$\bullet^5$ $\sqrt{0.6^2 - r^2}$ leading to $\tan a = \dfrac{r}{\sqrt{0.36 - r^2}}$	**1**
Notes: 1. $\bullet^5$ is not available unless 0.6^2 is explicitly seen.					
Commonly Observed Responses:					
		(ii)	$\bullet^6$ obtain second expression for $\tan a$ $\bullet^7$ equate expressions and form quadratic in r^2 $\bullet^8$ solve to find r	$\bullet^6$ $\dfrac{49}{9.8r}$ or $\dfrac{v^2}{gr}$ or $\dfrac{\frac{mv^2}{r}}{mg}$ $\bullet^7$ $\dfrac{r}{\sqrt{0.36 - r^2}} = \dfrac{49}{9.8r}$ leading to $r^4 + 25r^2 - 9 = 0$ $\bullet^8$ $r = 0.596\,\text{m}$	**3**
Notes: **1.** Accept 0.6m. 2. Do not award $\bullet^7$ and $\bullet^8$ if incorrect working leads to a **quadratic** in r **only**. 3. Mistake with signs in quartic: $r^4 - 25r^2 + 9 = 0$ leads to an answer of $r = 0.6044\,\text{m}$; $\bullet^8$ is available.					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
17.	(a)		$\bullet^1$ state expression $\bullet^2$ form equation and obtain one constant $\bullet^3$ obtain second constant and state partial fractions	$\bullet^1$ $\frac{A}{3+2x}+\frac{B}{3-2x}$ $\bullet^2$ $12=A(3-2x)+B(3+2x)$ and either $A=2$ or $B=2$ $\bullet^3$ $\frac{2}{3+2x}+\frac{2}{3-2x}$	**3**

Notes:

1. Accept $A=2$ and $B=2$ for $\bullet^3$ provided template seen in $\bullet^1$.
2. Do not award $\bullet^1$ for incorrect factorising, however $\bullet^2$ and $\bullet^3$ are still available provided a 'correct' template is used.
3. If an incorrect template has been used, award 0/3. Note, however, $\frac{A}{3+2x}+\frac{Bx+C}{3-2x}$, can be used (leads to $A=2$, $B=0$, $C=2$).

Commonly Observed Responses:

COR 1

Candidate uses $\frac{-12}{4x^2-9}$.

$\bullet^1$ $\frac{A}{2x+3}+\frac{B}{2x-3}$

$\bullet^2$ $-12=A(2x-3)+B(2x+3)$ and either $A=2$ or $B=-2$

$\bullet^3$ $\frac{2}{2x+3}-\frac{2}{2x-3}$

	(b)	(i)	$\bullet^4$ apply Newton's second law with substitution and complete	$\bullet^4$ $0.9-0.4v^2=1.2\frac{dv}{dt}$ leading to $\frac{dv}{dt}=\frac{9-4v^2}{12}$	**1**

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
17.	(b)	(ii)	$\bullet^5$ separate variables $\bullet^6$ substitute partial fractions $\bullet^7$ integrate $\bullet^8$ calculate constant of integration $\bullet^9$ calculate time	$\bullet^5$ $\int dt = \int \frac{12}{9-4v^2}\,dv$ $\bullet^6$ $\int dt = \int \frac{2}{3+2v} + \frac{2}{3-2v}\,dv$ $\bullet^7$ $t + c = \ln\lvert 3+2v\rvert - \ln\lvert 3-2v\rvert$ $\bullet^8$ $c = 0 \Rightarrow t = \ln\lvert 3+2v\rvert - \ln\lvert 3-2v\rvert$ $\bullet^9$ ln4 or 1.39 seconds	**5**

Notes:

1. For use of definite integration, $\bullet^8$ is awarded for the correct limits appearing at either $\bullet^6$ or $\bullet^7$.
2. If constant of integration is omitted, award $\bullet^7$ however $\bullet^8$ and $\bullet^9$ are unavailable.
3. If separated variables without the integral sign, but this is followed by a correct statement, treat as bad form.

Commonly Observed Responses:

COR 1

$v\frac{dv}{dx} = \frac{9-4x^2}{12}$

$\int v\,dv = \int \frac{9-4x^2}{12}\,dx$

$\frac{3}{4}x - \frac{x^3}{3} = \frac{v^2}{2} + c,$ when $v = 0,\ x = 0,\ c = 0$

Only $\bullet^5$ and $\bullet^8$ are available.

COR 2

Candidate **does not use** partial fractions to integrate and uses spurious integration techniques

eg $\int \frac{12}{9-4v^2}\,dv = \frac{12}{-8v}\ln\lvert 9-4v^2\rvert \ldots$ Only $\bullet^5$ is available.

COR 3

$\int dv = \int \frac{12}{9-4v^2}\,dt$

No marks available.

Question	Generic scheme	Illustrative scheme	Max mark

COR 4

Candidate starts with

$\frac{dv}{dt}=\frac{9-4v^2}{12}=\frac{3+2v}{2}+\frac{3-2v}{2}$ leading to

$\int dt=\int \frac{2}{3+2v}+\frac{2}{3-2v}dv$

Do not award $\bullet^5$ and $\bullet^6$, but $\bullet^7$, $\bullet^8$ and $\bullet^9$ are available.

COR 5

$\bullet^5$ $\frac{dt}{dv}=\frac{2}{3+2v}+\frac{2}{3-2v}$

$\bullet^6$ $\int dt=\int \frac{2}{3+2v}+\frac{2}{3-2v}dv$

$\bullet^7$ $t+c=\ln|3+2v|-\ln|3-2v|$

$\bullet^8$ $c=0\Rightarrow t=\ln|3+2v|-\ln|3-2v|$

$\bullet^9$ ln4 or 1.39 seconds

COR 6

Candidate uses exponentials. All marks available.

$\bullet^5$ $\int dt=\int \frac{12}{9-4v^2}dv$

$\bullet^6$ $\int dt=\int \frac{2}{3+2v}+\frac{2}{3-2v}dv$

$\bullet^7$ $\frac{3+2v}{3-2v}=e^{t+c}=Ae^t$

$\bullet^8$ $1=A\Rightarrow e^t=\frac{3+2v}{3-2v}$

$\bullet^9$ ln4 or 1.39 seconds

Question			Generic scheme	Illustrative scheme	Max mark
18.	(a)		•[1] find resultant force for any displacement •[2] show equation	•[1] e.g. $T_1 - T_2$ leading to $-\frac{\lambda x}{l} \times 2$ **OR** $T_1 + T_2$ leading to $\frac{\lambda x}{l} \times 2$ •[2] $\frac{-2\lambda x}{0.4} = 5\ddot{x} \Rightarrow \ddot{x} = -\lambda x$	**2**
Notes:					
Commonly Observed Responses: **COR 1** Candidate only uses one tension leading to $\frac{-\lambda x}{5} = \ddot{x}$. No marks available. **COR 2** Candidate differentiates SHM equation for displacement: $y = A\cos\omega t$ $\frac{dy}{dt} = -\omega A \sin \omega t$ $\frac{d^2y}{dx^2} = -\omega^2 A\cos\omega t \Rightarrow \ddot{x} = -\lambda x$ No marks available.					
	(b)		•[3] calculate ω •[4] calculate T	•[3] 8 •[4] $\frac{\pi}{4}$ or 0.785 seconds	**2**
Notes:					
Commonly Observed Responses: **COR 1** Candidate uses $\omega = \sqrt{\frac{\lambda}{ml}} \Rightarrow \omega = 4\sqrt{2}$; $T = \frac{2\pi}{\omega} = 1.11$ seconds. Do not award •[3], but •[4] is available for 1.11 seconds.					
	(c)		•[5] calculate maximum velocity	•[5] 0.48 ms^{-1}	**1**
Notes: 1. 0.339 ms^{-1} if $\omega = 4\sqrt{2}$ and 0.06 are used: •[5] is available.					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
18.	(d)		$\bullet^6$ calculate new value of ω $\bullet^7$ calculate mass	$\bullet^6$ $\frac{2\pi}{1.2}\left(=\frac{5\pi}{3}\right)$ $\bullet^7$ 11.67 kg	**2**

Notes:
1. Accept 12kg

Commonly Observed Responses:

COR 1

Candidate uses $\omega = \frac{5\pi}{3}$ with **repeated** use of $\omega = \sqrt{\frac{\lambda}{ml}} \Rightarrow m = 5.84\,\text{kg}$: $\bullet^7$ is available.

The same value for m comes from $F = mr\omega^2$, but this is not a valid strategy, so do not award $\bullet^7$.

COR 2

Candidate uses ω (instead of ω^2), leading to $m = 61.1\,\text{kg}$: only $\bullet^6$ is available

Question			Generic scheme	Illustrative scheme	Max mark
19.	(a)		$\bullet^1$ find centre of mass of triangle $\bullet^2$ find centre of mass of semicircle $\bullet^3$ take moments about A horizontally $\bullet^4$ find x-coordinate $\bullet^5$ take moments about A vertically $\bullet^6$ find y-coordinate	$\bullet^1$ (2, 6) $\bullet^2$ $9+\frac{4}{\pi}$ (10.27) $\bullet^3$ $14.137m\times 3+27m\times 2=41.137m\times\overline{x}$ stated or implied by $\bullet^4$ $\bullet^4$ $\overline{x}=2.34$ $\bullet^5$ $14.137m\times\left(9+\frac{4}{\pi}\right)+27m\times 6$ $=41.137m\times\overline{y}$ stated or implied by $\bullet^6$ $\bullet^6$ $\overline{y}=7.47$ So centre of mass is $(2.34,7.47)$	**6**

Notes:

1. Disregard the omission of m at $\bullet^3$ and $\bullet^5$.
2. $\bullet^6$ may be awarded if coordinates are stated individually.

Commonly Observed Responses:

COR 1

Shape	Area	Centre of mass
Semi-circle	$\frac{9\pi}{2}$	$\left(3,9+\frac{4}{\pi}\right)$
Triangle	27	(2, 6)
Lamina	$27+\frac{9\pi}{2}$	$(\overline{x},\overline{y})$

$\bullet^3$ and $\bullet^5$ can be implied by $\bullet^4$ and $\bullet^6$

<table>
<tr><th>Question</th><th>Generic scheme</th><th>Illustrative scheme</th><th>Max mark</th></tr>
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COR 2
Candidate calculates centre of mass for semi-circle incorrectly, $\bullet^2$ is unavailable.

Shape	Area	Centre of mass
Semi-circle	$\frac{9\pi}{2}$	$\left(3, \frac{4}{\pi}\right)$
Triangle	27	(2, 6)
Lamina	$27+\frac{9\pi}{2}$	$(\overline{x}, \overline{y})$

Centre of mass (2.34, 4.38) for mark $\bullet^6$

COR 3
Candidate takes moments about B.
Marks $\bullet^1$ and $\bullet^2$ can be awarded.

Shape	Area	Centre of mass
Semi-circle	$\frac{9\pi}{2}$	$\bullet^1\left(3, \frac{4}{\pi}\right)$
Triangle	27	$\bullet^2$ (2, -3)
Lamina	$27+\frac{9\pi}{2}$	$(\overline{x}, \overline{y})$

For (2.34, -1.53) mark $\bullet^6$ is unavailable (for mark $\bullet^6$ centre of mass must be from the origin A (2.34, 7.47)).

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<tr><td></td><td>(b)</td><td></td><td>$\bullet^7$ calculate angle</td><td>$\bullet^7$ 17.4°</td><td>1</td></tr>
<tr><td colspan="4">

Notes:

1. Follow through for use of $\tan^{-1}\left(\frac{\overline{x}}{\overline{y}}\right)$.

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Commonly Observed Responses:

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Question			Generic scheme	Illustrative scheme	Max mark
20.	(a)		$\bullet^1$ calculate the extension in the string $\bullet^2$ resolve vertically $\bullet^3$ calculate tension $\bullet^4$ determine λ	$\bullet^1$ 0.04 metres or equivalent $\bullet^2$ $2T\cos\theta = 60$ $\bullet^3$ 50 N $\bullet^4$ 325 N	**4**

Notes:

1. Extension and natural length can be either for the full string ($x = 0.04,\ l = 0.26$) or half of the string ($x = 0.02,\ l = 0.13$).

Commonly Observed Responses:

COR 1

Candidate uses $\frac{T}{2}$ consistent with their diagram: $\bullet^1$ and $\bullet^2$ are available, but do not award $\bullet^3$ for $T = 100\text{N}$ $\bullet^4$ is available for 650 N.

COR 2

Candidate uses $\frac{\lambda x}{l} = 60$

$\bullet^1$ is available, $\bullet^2$ and $\bullet^3$ are not available, but $\bullet^4$ is available for 390 N

	(b)	(i)	$\bullet^5$ calculate EPE at moment particle is released	$\bullet^5$ 144 J	**1**

Notes:

1. $\bullet^5$ can only be awarded if $x = 0.48$ is used with their λ.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
20.	(b)	(ii)	$\bullet^6$ calculate distance below PQ when string becomes slack	$\bullet^6$ 0.05 metres	**4**
			$\bullet^7$ use conservation of energy	$\bullet^7$ $\frac{1}{2}mv^2 + mgh = 144$	
			$\bullet^8$ determine KE at point string becomes slack (stated or implied by $\bullet^9$)	$\bullet^8$ 126 J	
			$\bullet^9$ calculate speed	$\bullet^9$ 6.42 ms^{-1}	

Notes:

1. Accept 6.4 or 6.41
2. Evidence of $\frac{\lambda x^2}{2l}$ must be seen in (b)(ii), or earlier, for marks $\bullet^7$, $\bullet^8$ and $\bullet^9$ to be available.
3. For $\bullet^7$, evidence of substitution of *their* elastic potential energy from (b)(i) must be seen.

Commonly Observed Responses:

COR 1

Candidate uses $h = 0.05$.

Do not award $\bullet^8$, but $\bullet^9$ is available for a speed of 6.79 ms^{-1}.

COR 2

Candidate uses $h = 0.35$

Do not award $\bullet^6$ and $\bullet^8$.

[END OF MARKING INSTRUCTIONS]