



National
Qualifications
2024

2024 Mathematics of Mechanics

Advanced Higher

Question Paper Finalised Marking Instructions

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General marking principles for Advanced Higher Mathematics of Mechanics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

The marking instructions for each question are generally in two sections:

generic scheme – this indicates why each mark is awarded

illustrative scheme – this covers methods which are commonly seen throughout the marking

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

- (a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.
- (b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.
- (c) One mark is available for each •. There are no half marks.
- (d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.
- (e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.
- (f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.
- (g) If an error is trivial, casual or insignificant, for example $6 \times 6 = 12$, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.
- (h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.

This is no longer a solution of a quadratic equation, so the mark is not awarded.

$$x^2 + 5x + 7 = 9x + 4$$

$$x - 4x + 3 = 0$$

$$x = 1$$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$x^2 + 5x + 7 = 9x + 4$$

$$x - 4x + 3 = 0$$

$$(x - 3)(x - 1) = 0$$

$$x = 1 \text{ or } 3$$

(i) Horizontal/vertical marking

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

$$\begin{array}{cc} \bullet^5 & \bullet^6 \\ \bullet^5 & x = 2 \quad x = -4 \\ \bullet^6 & y = 5 \quad y = -7 \end{array}$$

Horizontal: $\bullet^5 x = 2$ and $x = -4$ Vertical: $\bullet^5 x = 2$ and $y = 5$
 $\bullet^6 y = 5$ and $y = -7$ $\bullet^6 x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$$\begin{array}{ll} \frac{15}{12} \text{ must be simplified to } \frac{5}{4} \text{ or } 1\frac{1}{4} & \frac{43}{1} \text{ must be simplified to } 43 \\ \frac{15}{0.3} \text{ must be simplified to } 50 & \frac{4\cancel{5}}{3} \text{ must be simplified to } \frac{4}{15} \\ \sqrt{64} \text{ must be simplified to } 8^* & \end{array}$$

*The square root of perfect squares up to and including 144 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

$$\begin{aligned} & (x^3 + 2x^2 + 3x + 2)(2x + 1) \text{ written as} \\ & (x^3 + 2x^2 + 3x + 2) \times 2x + 1 \\ & = 2x^4 + 5x^3 + 8x^2 + 7x + 2 \\ & \text{gains full credit} \end{aligned}$$

- repeated error within a question, but not between questions or papers

(m) In any ‘Show that...’ question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

- (n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.
- (o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.
- (p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

| | |
|--|--|
| Strategy 1 attempt 1 is worth 3 marks. | Strategy 2 attempt 1 is worth 1 mark. |
| Strategy 1 attempt 2 is worth 4 marks. | Strategy 2 attempt 2 is worth 5 marks. |
| From the attempts using strategy 1, the resultant mark would be 3. | From the attempts using strategy 2, the resultant mark would be 1. |

In this case, award 3 marks.

Marking Instructions for each question

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|-----|--|---|--|----------|
| 1. | | | <ul style="list-style-type: none"> •¹ resolve vertically •² resolve horizontally •³ calculate θ •⁴ calculate T | <ul style="list-style-type: none"> •¹ $mg = 26 \cos 50^\circ + T \cos \theta$ •² $26 \sin 50^\circ = T \sin \theta$ •³ 17.9° •⁴ 64.8 N | 4 |
| Notes: | | | | | |
| Commonly Observed Responses: | | | | | |
| 2. | (a) | | <ul style="list-style-type: none"> •¹ state expression •² form linear equation and obtain one constant •³ find remaining constant and state full expression | <ul style="list-style-type: none"> •¹ $\frac{A}{2x-1} + \frac{B}{x+1}$ •² $7 - 2x = A(x+1) + B(2x-1)$ $A = 4$ or $B = -3$ •³ $\frac{4}{2x-1} - \frac{3}{x+1}$ | 3 |
| Notes: | | | | | |
| Do not accept $\frac{4}{2x-1} + \frac{-3}{x+1}$ | | | | | |
| Commonly Observed Responses: | | | | | |
| | (b) | | <ul style="list-style-type: none"> •⁴ integrate | <ul style="list-style-type: none"> •⁴ $2 \ln 2x-1 - 3 \ln x+1 + c$ | 1 |
| Notes: | | | | | |
| Do not penalise the omission of the constant of integration | | | | | |
| Commonly Observed Responses: | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|--|--|--|--|----------|
| 3. | | | <ul style="list-style-type: none"> •¹ find expression for momentum after collision •² apply conservation of momentum •³ calculate mass | <ul style="list-style-type: none"> •¹ $\frac{-30u}{3} + \frac{m_B u}{2}$ •² $30u = \frac{-30u}{3} + \frac{m_B u}{2}$ •³ 80 grams | 3 |
| Notes: 1. • ¹ may be implied by • ² . | | | | | |
| Commonly Observed Responses: | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|--|--|---|---|----------|
| 4. | | | <ul style="list-style-type: none"> •¹ start differentiation with evidence of use of quotient rule with denominator and one term of numerator correct •² complete differentiation •³ simplify answer | <ul style="list-style-type: none"> •¹ $\frac{3(1+x^2) - \dots}{(1+x^2)^2}$ or $\frac{\dots - 3x(2x)}{(1+x^2)^2}$ •² $\frac{3(1+x^2) - 3x(2x)}{(1+x^2)^2}$ •³ $\frac{3-3x^2}{(1+x^2)^2}$ | 3 |
| Notes: Do not award • ³ if there is incorrect working after a correct answer e.g. erroneous simplification of the algebraic fraction | | | | | |
| Commonly Observed Responses: | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|------------------------------|--|--|--|--|----------|
| 5. | | | <ul style="list-style-type: none"> •¹ create equation for maximum speed or acceleration •² create second equation and divide to find ω •³ calculate amplitude •⁴ calculate speed | <ul style="list-style-type: none"> •¹ $a\omega^2 = 20$ or $a\omega = 10$ •² $a\omega = 10$ or $a\omega^2 = 20$ and $\omega = 2\text{rad s}^{-1}$ •³ 5 metres •⁴ $\sqrt{96}$ or 9.80ms^{-1} | 4 |
| Notes: | | | | | |
| Commonly Observed Responses: | | | | | |

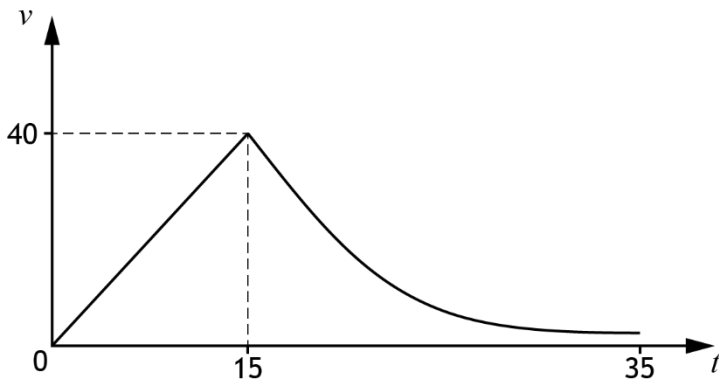
| Question | | | Generic scheme | Illustrative scheme | Max mark |
|--|--|--|--|---|----------|
| 6. | | | <ul style="list-style-type: none"> •¹ start to use chain rule •² complete chain rule •³ evaluate | <ul style="list-style-type: none"> •¹ $2\operatorname{cosec}(3x) \times \dots$ •² $2\operatorname{cosec}(3x) \times (-\operatorname{cosec}(3x)\cot(3x)) \times 3$ •³ 12 | 3 |
| Notes: | | | | | |
| Commonly Observed Responses: | | | | | |
| 7. | | | <ul style="list-style-type: none"> •¹ take moments about any point •² equate to moments in opposite direction •³ calculate reaction force at Q | <ul style="list-style-type: none"> •¹ eg $22g \times 1.5 + 45g \times 2$ or $3R_Q$ •² eg $22g \times 1.5 + 45g \times 2 = 3R_Q$ •³ 41g or 401.8 N | 3 |
| Notes: | | | | | |
| • ³ is unavailable if g is absent by • ² stage | | | | | |
| Commonly Observed Responses: | | | | | |
| Alternative method | | | | | |
| | | | <ul style="list-style-type: none"> •¹ take moments about any point •² resolve forces vertically •³ calculate reaction force at Q | <ul style="list-style-type: none"> •¹ eg $22g \times 2.5 + 45g \times 3 = R_P + 4R_Q$ •² eg $R_P + R_Q = 22g + 45g$ •³ 41g or 401.8 N | 3 |
| Notes: | | | | | |
| Commonly Observed Responses: | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|------------------------------|-----|--|--|--|----------|
| 8. | (a) | | <ul style="list-style-type: none"> •¹ find expression for time •² find expression for height •³ substitute expression for time and simplify to required form | <ul style="list-style-type: none"> •¹ $t = \frac{x}{u \cos \theta}$ •² $u \sin \theta \times t - \frac{1}{2} g t^2$ •³ $u \sin \theta \times \frac{x}{u \cos \theta} - \frac{1}{2} g \frac{x^2}{u^2 \cos^2 \theta}$ leading to $y = x \tan \theta - \frac{g x^2}{2 u^2 \cos^2 \theta}$ | 3 |
| Notes: | | | | | |
| Commonly Observed Responses: | | | | | |
| | (b) | | <ul style="list-style-type: none"> •⁴ substitute into trajectory equation •⁵ set up quadratic equation •⁶ solve for $\tan \theta$ •⁷ give range of angles | <ul style="list-style-type: none"> •⁴ $9 = 30 \tan \theta - \frac{9.8 \times 30^2}{2 \times 20^2} (1 + \tan^2 \theta)$ •⁵ eg $11.025 \tan^2 \theta - 30 \tan \theta + 20.025 = 0$ •⁶ $\tan \theta = 1.174$ or 1.546 •⁷ $49.6^\circ < \theta < 57.1^\circ$ | 4 |
| Notes: | | | | | |
| Commonly Observed Responses: | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|------------------------------|--|--|--|---|----------|
| 9. | | | <ul style="list-style-type: none"> •¹ differentiate implicitly with respect to t •² start to differentiate using product rule •³ complete differentiation •⁴ determine value of t when $v = 0$ •⁵ evaluate instantaneous acceleration | <ul style="list-style-type: none"> •¹ $3 \frac{dv}{dt} + \dots$ •² $2te^v + \dots$ •³ $3 \frac{dv}{dt} + 2te^v + t^2 e^v \frac{dv}{dt} = 0$ •⁴ $t = 3$ •⁵ $-\frac{1}{2} \text{ ms}^{-2}$ | 5 |
| Notes: | | | | | |
| Commonly Observed Responses: | | | | | |
| Alternative method | | | | | |
| | | | <ul style="list-style-type: none"> •¹ start to differentiate implicitly with respect to v using product rule •² complete product rule •³ complete differentiation •⁴ determine value of t when $v = 0$ •⁵ evaluate instantaneous acceleration | <ul style="list-style-type: none"> •¹ $2t \frac{dt}{dv} e^v$ •² $t^2 e^v$ •³ $3 + 2t \frac{dt}{dv} e^v + t^2 e^v = 0$ •⁴ $t = 3$ •⁵ $-\frac{1}{2} \text{ ms}^{-2}$ | 5 |
| Notes: | | | | | |
| Commonly Observed Responses: | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|------------------------------|--|--|---|---|----------|
| 10. | | | <ul style="list-style-type: none"> •¹ consider energy at top •² consider energy at bottom and use conservation of energy. •³ determine the angle •⁴ determine the speed | <ul style="list-style-type: none"> •¹ $mgr(1 - \cos \theta)$ •² $\frac{1}{2}mv^2 = mgr(1 - \cos \theta)$ •³ 80.2° (1.4 radians) •⁴ 4.9ms^{-1} | 4 |
| Notes: | | | | | |
| Commonly Observed Responses: | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|--|--|--|--|----------|
| 11. | | | <ul style="list-style-type: none"> •¹ find integrating factor •² multiply by integrating factor and state equation •³ integrate and include constant of integration •⁴ find constant and state particular solution in correct form | <ul style="list-style-type: none"> •¹ e^{-2x} •² $\frac{d}{dx}(e^{-2x}y) = 3$ •³ $e^{-2x}y = 3x + c$ •⁴ $y = 3xe^{2x} + 5e^{2x}$ or equivalent | 4 |
| Notes: 1. Only • ¹ and • ² available if c is omitted. | | | | | |
| Commonly Observed Responses: | | | | | |

| Question | | Generic scheme | Illustrative scheme | Max mark |
|--|-----|---|--|----------|
| 12. | (a) | <ul style="list-style-type: none"> •¹ correct shape of graph •² all correct annotations  | | 2 |
| Notes: | | | | |
| Commonly Observed Responses: | | | | |
| | (b) | <ul style="list-style-type: none"> •³ set up integral •⁴ integrate •⁵ evaluate integral •⁶ hence determine total distance travelled | <ul style="list-style-type: none"> •³ $\int_{15}^{35} 905e^{-0.20793t} dt$ •⁴ $\left[-4352.4e^{-0.20793t} \right]_{15}^{35}$ •⁵ 189 •⁶ distance = $\frac{1}{2} \times 15 \times 40 + 189$ = 489m | 4 |
| Notes: | | | | |
| At • ³ do not penalise the omission of dt | | | | |
| Commonly Observed Responses: | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|--|--|--|---|--|----------|
| 13. | | | <ul style="list-style-type: none"> •¹ separate the variables •² integrate •³ determine the value of the constant of integration •⁴ calculate velocity | <ul style="list-style-type: none"> •¹ $\int \frac{1}{v} dv = \int \frac{2}{1+t} dt$ •² $\ln v = 2 \ln(1+t) + c$ •³ $\ln 2$ •⁴ 32 ms^{-1} | 4 |
| Notes: 1. If constant of integration is omitted at • ² , marks • ³ and • ⁴ are unavailable. 2. Do not award • ¹ if either dv or dt or both are omitted 3. Where a candidate attempts to integrate an expression involving v with respect to t , or vice versa, award 0/4 | | | | | |
| Commonly Observed Responses: | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|-----|--|---|--|----------|
| 14. | (a) | | • ¹ state integral | • ¹ $\sec 3x + c$ | 1 |
| | (b) | | • ² start integration by parts | • ² $\sin^2 3x \sec 3x$ | 2 |
| | | | • ³ complete integration | • ³ $\sin^2 3x \sec 3x + 2 \cos 3x + c$ | |
| Notes: 1. Do not withhold • ¹ or • ³ for the omission of the constant of integration. | | | | | |
| Commonly Observed Responses: | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|------------------------------|-----|--|--|---|----------|
| 15. | (a) | | • ¹ calculate distance | • ¹ 10 metres | 1 |
| | (b) | | • ² calculate velocity of car after 5 seconds • ³ obtain expression for displacement of car or motorbike • ⁴ obtain second expression and equate • ⁵ calculate time | • ² 4ms^{-1} • ³ $0.9t^2$ or $0.4t^2 + 4t + 10$ • ⁴ $0.9t^2 = 0.4t^2 + 4t + 10$ • ⁵ 10 seconds | 4 |
| Notes: | | | | | |
| Commonly Observed Responses: | | | | | |
| Alternative method 1 | | | | | |
| | | | • ² obtain expression for displacement of car or motorbike • ³ obtain second expression for displacement • ⁴ equate expressions and start to solve • ⁵ calculate time | • ² $0.9t^2$ or $0.4(t+5)^2$ • ³ $0.4(t+5)^2$ or $0.9t^2$ • ⁴ $0.9t^2 = 0.4t^2 + 4t + 10$ • ⁵ 10 seconds | 4 |
| Notes: | | | | | |
| Commonly Observed Responses: | | | | | |
| Alternative method 2 | | | | | |
| | | | • ² obtain expression for displacement of car or motorbike • ³ obtain second expression for displacement • ⁴ equate expressions and start to solve • ⁵ calculate time | • ² $0.4t^2$ or $0.9(t-5)^2$ • ³ $0.9(t-5)^2$ or $0.4t^2$ • ⁴ $0.4t^2 = 0.9t^2 - 9t + 22.5$ • ⁵ $t = 15$ leading to 10 seconds | 4 |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|--|-----|--|---|---|----------|
| 15. | (c) | | <ul style="list-style-type: none"> •⁶ find velocity of car after 15.8 secs •⁷ find the distance car has to decelerate •⁸ calculate deceleration | <ul style="list-style-type: none"> •⁶ 12.64 ms^{-1} •⁷ 190.144m •⁸ 0.42 ms^{-2} | 3 |
| Notes: For • ⁸ accept $a = -0.42 \text{ ms}^{-2}$ | | | | | |
| Commonly Observed Responses: | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|--|--|--|--|---|----------|
| 16. | | | <ul style="list-style-type: none"> •¹ find $\frac{dx}{dt}$ or $\frac{dy}{dt}$ •² find $\frac{dy}{dx}$ •³ solve for t •⁴ find coordinates | <ul style="list-style-type: none"> •¹ $\frac{dx}{dt} = 3e^{3t} - 2e^{2t}$ or $\frac{dy}{dt} = 3e^{3t} + 2e^{2t}$ •² $\frac{3e^{3t} + 2e^{2t}}{3e^{3t} - 2e^{2t}}$ stated or implied by •³ $\ln 2$ •⁴ (4,12) | 4 |
| Notes: For • ⁴ accept $x = 4, y = 12$ | | | | | |
| Commonly Observed Responses: | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|---|--|--|---|--|----------|
| 17. | | | <ul style="list-style-type: none"> •¹ calculate ω •² apply Newton's inverse law of gravitation at the surface of the planet •³ apply Newton's inverse law of gravitation at the satellite and equate with angular acceleration •⁴ combine equations and substitute value of ω •⁵ rearrange to the required result | <ul style="list-style-type: none"> •¹ $\frac{1}{1000}$ •² $a = \frac{GM}{R^2}$ •³ $\frac{GM}{(pR)^2} = \omega^2 pR$ •⁴ $\frac{aR^2}{(pR)^2} = \left(\frac{1}{1000}\right)^2 pR$ •⁵ $\frac{a}{p^2} = \frac{1}{1000^2} pR$ leading to $R = \frac{1000^2 a}{p^3}$ | 5 |
| Notes: Accept the use of k instead of GM at • ² and • ³ | | | | | |
| Commonly Observed Responses: | | | | | |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|----------|-----|--|--|--|----------|
| 18. | (a) | | <ul style="list-style-type: none"> •¹ resolve forces perpendicular to plane for full box •² resolve forces parallel to plane for full box •³ resolve forces parallel to plane for empty box •⁴ equate P to 5Q •⁵ rearrange and calculate value of μ | <ul style="list-style-type: none"> •¹ $R = 60g \cos 10^\circ$ •² $P = 60g \sin 10^\circ + \mu R$ •³ $Q + 40g \sin 10^\circ = \mu(40g \cos 10^\circ)$ •⁴ $60g(\sin 10^\circ + \mu \cos 10^\circ) = 5(40g(\mu \cos 10^\circ - \sin 10^\circ))$ •⁵ $\mu = 0.327$ | 5 |

Notes:

1. Appropriate working must appear after •⁴ has been awarded before the award of •⁵.

Commonly Observed Responses:

| | | | | | |
|--|-----|--|--|---|---|
| | (b) | | <ul style="list-style-type: none"> •⁶ set up equation of motion for system under tension •⁷ calculate acceleration for system under tension •⁸ calculate distance travelled by boxes before cable snaps •⁹ calculate velocity of boxes at the point the cable snaps •¹⁰ calculate acceleration for system moving under gravity •¹¹ calculate remaining distance travelled and total distance | <ul style="list-style-type: none"> •⁶ $300 - 60g(\mu \cos 10^\circ + \sin 10^\circ) = 60a$ •⁷ $a = 0.142$ •⁸ $s = 7.1$ •⁹ $v = 1.42$ •¹⁰ $a = -4.86$ •¹¹ 7.31 metres | 6 |
|--|-----|--|--|---|---|

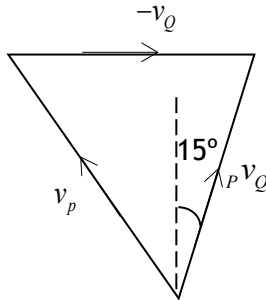
Notes:

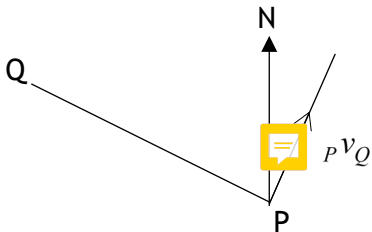
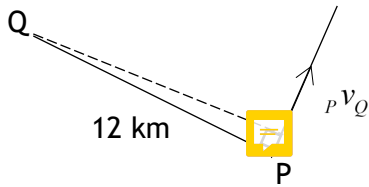
1. At •¹¹, accept any answer that rounds to 7.3.

Commonly Observed Responses:

Values with exact coefficient of friction are as follows:

$a = 0.137\dots$, $s = 6.89\dots$, $v = 1.37\dots$, $a = -4.86\dots$, $7.08\dots$

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|--------------------|-----|------|--|--|----------|
| 19. | (a) | (i) | <ul style="list-style-type: none"> •¹ interpret given information •² find vector for velocity of P •³ find speed of P | <ul style="list-style-type: none"> •¹ $\mathbf{v}_Q = \begin{pmatrix} -18 \\ 0 \end{pmatrix}$ ${}_P\mathbf{v}_Q = \begin{pmatrix} 20 \sin 15^\circ \\ 20 \cos 15^\circ \end{pmatrix}$ •² $\begin{pmatrix} -12.8 \\ 19.3 \end{pmatrix}$ •³ 23.2 kmh^{-1} | 3 |
| | | (ii) | <ul style="list-style-type: none"> •⁴ find direction of P | <ul style="list-style-type: none"> •⁴ Bearing: 326.4° | 1 |
| Alternative method | | | | | |
| | (a) | (i) | <ul style="list-style-type: none"> •¹ construct triangle with Q brought to rest •² use cosine rule to start to find magnitude of vector for velocity of P •³ substitute correct angle of 75° and calculate speed | <ul style="list-style-type: none"> •¹  •² $\sqrt{18^2 + 20^2 - 2 \times 18 \times 20 \times \cos \dots}$ •³ 23.2 kmh^{-1} | 3 |
| | | (ii) | <ul style="list-style-type: none"> •⁴ find direction of P | <ul style="list-style-type: none"> •⁴ Bearing: 326.4° | 1 |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|----------------------|-----|--|---|---|----------|
| 19. | (b) | | <ul style="list-style-type: none"> •⁵ express displacement for both boats after t hours •⁶ state and simplify expression for relative displacement •⁷ use appropriate method to minimise displacement •⁸ differentiate and equate to zero to minimise displacement •⁹ interpret answer to give time at which they are closest. | <ul style="list-style-type: none"> •⁵ $\mathbf{r}_P = \begin{pmatrix} -12.8t \\ 19.3t \end{pmatrix}$ $\mathbf{r}_Q = \begin{pmatrix} -12 \sin 70^\circ - 18t \\ 12 \cos 70^\circ \end{pmatrix}$ •⁶ ${}_Q\mathbf{r}_P = \begin{pmatrix} -5.2t - 12 \sin 70^\circ \\ 12 \cos 70^\circ - 19.3t \end{pmatrix}$ •⁷ ${}_Q\mathbf{r}_P ^2 = (-5.2t - 12 \sin 70^\circ)^2 + (12 \cos 70^\circ - 19.3t)^2$ •⁸ $\frac{d}{dt} {}_Q\mathbf{r}_P ^2 = 799.06t - 41.15 = 0$ •⁹ 12:03 pm | 5 |
| Alternative method 1 | | | | | |
| | | | <ul style="list-style-type: none"> •⁵ assemble facts and know to use PQ and ${}_P\mathbf{v}_Q$ •⁶ establish suitable right-angled triangle for closest approach •⁷ calculate angle at P •⁸ calculate closest approach •⁹ state time | <ul style="list-style-type: none"> •⁵  •⁶  •⁷ 85° •⁸ 1.05 km •⁹ 12:03 pm | 5 |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|----------------------|-----|--|--|---|----------|
| 19. | (b) | | (continued) | | |
| Alternative method 2 | | | | | |
| | | | <ul style="list-style-type: none">•⁵ express displacement for both boats after t hours•⁶ state and simplify expression for relative displacement•⁷ use appropriate method to minimise displacement•⁸ find expression for scalar product•⁹ interpret answer to give time at which they are closest. | <ul style="list-style-type: none">•⁵$\mathbf{r}_P = \begin{pmatrix} -12.8t \\ 19.3t \end{pmatrix}$•⁶$\mathbf{r}_Q = \begin{pmatrix} -12 \sin 70^\circ - 18t \\ 12 \cos 70^\circ \end{pmatrix}$•⁶${}_Q\mathbf{r}_P = \begin{pmatrix} -5.2t - 12 \sin 70^\circ \\ 12 \cos 70^\circ - 19.3t \end{pmatrix}$•⁷ eg ${}_Q\mathbf{r}_P \cdot {}_P\mathbf{v}_Q = 0$•⁸$-399.76t + 20.92 = 0$•⁹ 12:03 pm | 5 |

| Question | | | Generic scheme | Illustrative scheme | Max mark |
|------------------------------|-----|--|---|---|----------|
| 20. | (a) | | <ul style="list-style-type: none"> •¹ apply Newton's second law with substitution for tractive force •² substitute for normal reaction force and obtain expression | <ul style="list-style-type: none"> •¹ $\frac{P}{V} - \mu R = ma$ •² $\frac{P}{V} - \mu mg = ma$ leading to $P = mV(a + 0.1g)$ | 2 |
| | (b) | | <ul style="list-style-type: none"> •³ apply Newton's second law parallel to the slope •⁴ resolve perpendicular to the slope •⁵ combine equations and substitute previous expression •⁶ calculate acceleration | <ul style="list-style-type: none"> •³ $\frac{3P}{V} - \mu R - mg \sin \theta = ma$ •⁴ $R = mg \cos \theta$ •⁵ $\frac{3mV(a + \mu g)}{V} - \mu mg \cos \theta - mg \sin \theta = ma$ •⁶ 1.40 ms^{-2} | 4 |
| Notes: | | | | | |
| Commonly Observed Responses: | | | | | |

[END OF MARKING INSTRUCTIONS]