National
Qualifications
2022

2022 Mathematics of Mechanics

Advanced Higher

Finalised Marking Instructions

General marking principles for Advanced Higher Mathematics of Mechanics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example 6 × 6 = 12, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.	$x^2+5x+7=9x+4$ $x-4x+3=0$
This is no longer a solution of a quadratic equation, so the mark is not awarded.	$x=1$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.	$x^2+5x+7=9x+4$ $x-4x+3=0$ $(x-3)(x-1)=0$ $x=1$ or 3

(i) Horizontal/vertical marking
If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x = 2$	$x = -4$
$\bullet^6$	$y = 5$	$y = -7$

Horizontal: $\bullet^5$ $x = 2$ and $x = -4$
$\bullet^6$ $y = 5$ and $y = -7$

Vertical: $\bullet^5$ $x = 2$ and $y = 5$
$\bullet^6$ $x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\qquad$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0{\cdot}3}$ must be simplified to 50 $\qquad$ $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 144 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $\left(x^3+2x^2+3x+2\right)\left(2x+1\right)$ written as
  $\left(x^3+2x^2+3x+2\right)\times 2x+1$
  $=2x^4+5x^3+8x^2+7x+2$
  gains full credit

- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Marking Instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.	(a)		•[1] calculate impulse •[2] calculate speed	•[1] 78 •[2] 9.75	2
Notes: 1. Do not award •[2] for a negative answer					
			Alternative Solution •[1] calculate acceleration •[2] calculate speed	•[1] 8.125 •[2] 9.75	
	(b)		•[3] state velocity after impact •[4] calculate impulse on object	•[3] −9.75 •[4] 156	2
Notes: 1. •[3] can be implied in •[4] 2. Do not award •[4] for a negative answer. However treat negative answers for both •[2] and •[4] as a repeated error.					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
2.			•1 state expression	•1 $\frac{A}{1+x}+\frac{B}{1-x}+\frac{C}{(1-x)^2}$	3
			•2 form equation and find one unknown	•2 $2-3x-x^2=A(1-x)^2$ $+B(1-x)(1+x)+C(1+x)$ **and one from** $A=1, B=2, C=-1$	
			•3 obtain remaining unknowns	•3 remaining from $A=1, B=2, C=-1$	

Notes:

1. Evidence for •1 may appear at •3

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
3.	(a)		•¹ substitute values into equation of motion •² calculate maximum height	•¹ $0=(25\sin 30)^2-2gs$ •² 7.97	**2**
			Alternative Solution •¹ calculate time to maximum height •² calculate speed	 •¹ 1.28 •² 9.75	
Notes:					
	(b)		•³ substitute values into equation of motion •⁴ solve quadratic for t •⁵ calculate the horizontal distance	•³ $1=(25\sin 30)t-\frac{1}{2}gt^2$ •⁴ 2.47 •⁵ 53.4	**3**
Notes:					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
4.	(a)		•[1] state the period of the motion	•[1] 16	1
	(b)		•[2] calculate value of ω •[3] calculate amplitude	•[2] $\frac{\pi}{8}$ •[3] 15.3	2

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
5.			•¹ solve auxiliary equation	•¹ $m = 2$, $m = -3$	5
			•² state general solution	•² $x = Ae^{2t} + Be^{-3t}$	
			•³ differentiate	•³ $\frac{dx}{dt} = 2Ae^{2t} - 3Be^{-3t}$	
			•⁴ form equations and solve for one constant	•⁴ $A = 1$ or $B = -1$	
			•⁵ find second constant and state particular solution	•⁵ $x = e^{2t} - e^{-3t}$	

Notes:

1. •¹ may be implied by •²

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
6.	(a)		•¹ integrate a with respect to t and include constant of integration •² use initial conditions and complete	•¹ $v = \int a\,dt = at + c$ •² $t = 0, v = u \Rightarrow c = u$ $v = u + at$	2
			Alternative Solution •¹ set up integral and include limits •² integrate and complete	•¹ $\int_u^v dv = \int_0^t a\,dt$ •² $v - u = at - 0$ $v = u + at$	
Notes:					
	(b)		•³ Integrate expression from part a •⁴ find constant and state expression	•³ $\int v\,dt = ut + \frac{1}{2}at^2 + k$ •⁴ $t = 0 \;\; s = 0 \Rightarrow k = 0$ $s = ut + \frac{1}{2}at^2$	2
Notes: • Do not penalise the omission of a constant for •³, however •⁴ is then unavailable • Accept use of the same letter for the constant in parts (a) and (b)					
			Alternative Solution •¹ set up integral and include limits •² integrate and complete	•¹ $\int_0^s ds = \int_0^t (u + at)\,dt$ •² $s = ut + \frac{1}{2}at^2$	

Question			Generic scheme	Illustrative scheme	Max mark
7.			•¹ choose functions to differentiate and integrate and start to integrate •² continue to integrate •³ complete integration, simplify and include constant of integration	•¹ $18x\left(-\frac{1}{3}\cos 3x\right)-\int\ldots$ •² $\ldots-\int\left(18\times\left(-\frac{1}{3}\cos 3x\right)\right)dx$ •³ $-6x\cos 3x+2\sin 3x+c$	3

Notes:

Do not withhold •³ if constant of integration is omitted

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
8.			•1 resolve forces vertically •2 apply Newton's second law horizontally •3 combine equations •4 convert angular speed to radians per second •5 substitute values and calculate	•1 $R = mg$ •2 $\mu R = mr\omega^2$ •3 $\mu mg = mr\omega^2$ •4 3π •5 0.634	**5**

Notes:

1. Accept the use of $\frac{v^2}{r}$ instead of $r\omega^2$ at •2 and •3

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
9.			•¹ know that volume $=\int \pi y^2 dx$ and begin to substitute	•¹ $\int \pi y^2 dx = \int \pi \frac{\dots}{\left(3x^3-1\right)} dx$	**6**
			•² complete substitution and introduce limits	•² $\int_1^2 \pi \frac{36x^2}{\left(3x^3-1\right)} dx$	
			•³ differentiate	•³ $du = 9x^2 dx$ **or** $\frac{du}{dx} = 9x^2$	
			•⁴ determ\`ine limits	•⁴ $\int_2^{23} \dots du$	
			•⁵ complete integral	•⁵ $4\pi \int_2^{23} \frac{1}{u^2} du$	
			•⁶ integrate and evaluate	•⁶ $\frac{42\pi}{23}$	
Notes: (see next page)					

Question			Generic scheme	Illustrative scheme	Max mark
9.			**Alternative Solution (without calculating limits for u)**		
			•¹ know that volume $=\int \pi y^2 dx$ and begin to substitute	•¹ $\int \pi y^2 dx = \int \pi \frac{\cdots}{\left(3x^3-1\right)} dx$	
			•² complete substitution and introduce limits	•² $\int_1^2 \pi \frac{36x^2}{(3x^3-1)^2} dx$	
			•³ differentiate	•³ $du = 9x^2 dx$ **or** $\frac{du}{dx} = 9x^2$	
			•⁴ state integral	•⁴ $4\pi \int_{\ldots}^{\ldots} \frac{1}{u^2} du$	
			•⁵ integrate and include limits	•⁵ $4\pi \left[\frac{-1}{3x^3-1} \right]_1^2$	
			•⁶ integrate and evaluate	•⁶ $\frac{42\pi}{23}$	

Notes:

1. For •¹ dx must appear prior to •³
2. For •² to be awarded, correct limits must be present. Evidence of this may appear elsewhere
3. •¹ may also be awarded for $\int_1^2 \pi y^2 dx$
4. •⁶ is unavailable if the limits 1 and 2 are substituted for u
5. Treat the appearance of the limits 1 and 2 at •⁴ in 1st method as bad form if it is later corrected

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
10.	(a)		•[1] consider total energy at A •[2] use conservation of energy to find speed at B	•[1] $=0.1\times9.8\times0.6+\frac{1}{2}\times0.1\times1.2^2$ •[2] 3.63	2

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•[3] state the force equation when rod is at A •[4] calculate tension •[5] identify as thrust/compression	•[3] $T-mg\cos180°=\frac{mv^2}{r}$ •[4] −0.5 •[5] rod is in thrust/compression	3

Notes:

1. If a positive answer is awarded •[4] as a follow through, do not award •[5] for "rod is in tension"

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(c)		•[6] consider forces in equilibrium when tension is zero. •[7] consider potential energy at this point. •[8] use conservation of energy to find kinetic energy at angle θ •[9] combine equations to eliminate v •[10] solve for θ	•[6] $\frac{mv^2}{r}+mg\cos\theta=0$ •[7] $E_P=mgr(1-\cos\theta)$ •[8] $\frac{1}{2}mv^2=\frac{1}{2}mv_B{}^2-E_P$ •[9] $\frac{m}{r}\left(v_B{}^2-2gr(1-\cos\theta)\right)+mg\cos\theta=0$ •[10] 146°	5

Notes:

1. Evidence for •[6] may appear later in the solution
2. For •[10] accept 2.55 radians

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
11.			•¹ identify integral form of integrating factor	•¹ $e^{\int -\frac{1}{x}dx}$	**6**
			•² determine integrating factor	•² $\frac{1}{x}$	
			•³ write as integral equation	•³ $\frac{1}{x}y = \int e^{2x}dx$	
			•⁴ integrate	•⁴ $\frac{1}{x}y = \frac{1}{2}e^{2x} + c$	
			•⁵ evaluate constant	•⁵ $c = e^2$	
			•⁶ form particular solution	•⁶ $y = \frac{1}{2}xe^{2x} + e^2x$	

Notes:

1. If constant of integration is omitted at •⁴, award •⁴ but •⁵ and •⁶ are unavailable

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
12.	(a)		•[1] resolve forces perpendicular to the plane •[2] resolve forces parallel to the plane for μR acting up the plane •[3] strategy to eliminate R and substitute for F •[4] simplify by eliminating *mg* and fractions •[5] use $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and rearrange to required answer	•[1] $R + F\sin\theta = mg\cos\theta$ •[2] $\mu R + F\cos\theta = mg\sin\theta$ •[3] $\mu\left(mg\cos\theta - \frac{1}{2}mg\sin\theta\right) + \frac{1}{2}mg\cos\theta = mg\sin\theta$ •[4] $2\mu\cos\theta - \mu\sin\theta + \cos\theta = 2\sin\theta$ •[5] working legitimately leading to $\mu = \dfrac{2\tan\theta - 1}{2 - \tan\theta}$	**5**

Notes:

1. W may be used instead of *mg*.
2. •[3] is unavailable if $R = mg$ is stated at •[1]

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
12.	(b)		**Method 1**	**Method 1**	5
			•6 determine the value of μ	•6 0.109	
			•7 resolve forces parallel to the slope	•7 $kmg\cos 30° = 0.109R + mg\sin 30°$	
			•8 resolve forces perpendicular to the slope	•8 $R + kmg\sin 30° = mg\cos 30°$	
			•9 substitute for R	•9 $kmg\cos 30° = 0.109(mg\cos 30° - kmg\sin 30°) + mg\sin 30°$	
			•10 state magnitude of force in terms of m and g	•10 $0.646mg$	
			Method 2	**Method 2**	
			•6 determine the value of μ	•6 0.109	
			•7 resolve forces parallel to the slope	•7 $F\cos 30° = 0.109R + mg\sin 30°$	
			•8 resolve forces perpendicular to the slope	•8 $R + F\sin 30° = mg\cos 30°$	
			•9 substitute for R	•9 $F\cos 30° = 0.109(mg\cos 30° - F\sin 30°) + mg\sin 30°$	
			•10 state magnitude of force	•10 $0.646mg$	

Notes:

1. •9 is unavailable if $R = mg$ is stated at •8

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
13.	(a)		•1 evidence of use of quotient rule with denominator and one term in numerator correct •2 complete differentiation •3 simplify and complete	•1 $\frac{(\tan x+1)\sec x\tan x-\ldots}{(\tan x+1)^2}$ or $\frac{\ldots-\sec x\sec^2 x}{(\tan x+1)^2}$ •2 $\frac{(\tan x+1)\sec x\tan x-\sec x\sec^2 x}{(\tan x+1)^2}$ $\ldots=\frac{\sec x(\tan x-1)}{(\tan x+1)^2}$ leading to **either** $f(x)\frac{\tan x-1}{\tan x+1}$ **or** $\frac{\sec x}{\tan x+1}\cdot\frac{\tan x-1}{\tan x+1}$	3

Notes:

1. For •3 to be awarded, the use of $1+\tan^2 x=\sec^2 x$ or equivalent should be obvious

	(b)		•4 recognise form of integral •5 integrate	•4 $\int\frac{f'(x)}{f(x)}dx$ **stated or implied by •5** •5 $\ln\left\lvert\frac{\sec x}{\tan x+1}\right\rvert+c$	2

Notes:

1. Accept $\ln\left(\frac{\sec x}{\tan x+1}\right)+c$
2. Do not penalise the omission of the constant of integration

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
14.	(a)		•¹ state condition for maximum velocity with substitution •² solve the equation for positive value of x.	•¹ $15+x-2x^2=0$ •² 3	**2**
	(b)	(i)	•³ set up integral for work done •⁴ integrate •⁵ calculate the work done	•³ $\int\left(75+5x-10x^2\right)dx$ •⁴ $\left[75x+\frac{5}{2}x-\frac{10}{3}x^2\right]_0^3$ •⁵ 157.5	**3**
		(ii)	•⁶ use the work-energy principle •⁷ determine value of	•⁶ $\frac{1}{2}\times5\times v^2-\frac{1}{2}\times5\times0^2=157.5$ •⁷ 7.94 **or** $\sqrt{63}$	**2**
			Alternative Solution •⁶ replace acceleration with $v\frac{dv}{dx}$, separate variables and set up integration •⁷ integrate and complete	$\int v\,dv=\int\left(15+x-2x^2\right)dx$ **or** $\int_0^v v\,dv=\int_0^3\left(15+x-2x^2\right)dx$ •⁷ 7.94 **or** $\sqrt{63}$	

Notes:

1. If an indefinite integral is used in the alternative solution, a constant of integration must be considered for •⁷ to be awarded

Question			Generic scheme	Illustrative scheme	Max mark
15.	(a)	(i)	•[1] obtain velocity vector •[2] substitute and calculate speed	•[1] $\begin{pmatrix} 3000t+240 \\ 0 \\ 80t+50 \end{pmatrix}$ •[2] 843	**2**
			Alternative Solution substitute into equation of motion •[2] calculate speed	•[1] $\begin{pmatrix} 240 \\ 0 \\ 50 \end{pmatrix} + \begin{pmatrix} 3000 \\ 0 \\ 80 \end{pmatrix} \times 0.2$ •[2] 843	
Notes: Do not penalise candidates who use two-dimensional vectors in (a)(i)					
		(ii)	•[3] obtain position vector •[4] evaluate position vector	•[3] $\begin{pmatrix} 1500t^2+240t \\ 0 \\ 40t^2+50t \end{pmatrix}$ •[4] $\begin{pmatrix} 108 \\ 0 \\ 11.6 \end{pmatrix}$	**2**
			Alternative Solution •[1] substitute into equation of motion •[2] evaluate position vector	•[1] $\begin{pmatrix} 240 \\ 0 \\ 50 \end{pmatrix} + \begin{pmatrix} 3000 \\ 0 \\ 80 \end{pmatrix} \times 0.2$ •[2] $\begin{pmatrix} 108 \\ 0 \\ 11.6 \end{pmatrix}$	
Notes: Do not penalise candidates who use two-dimensional vectors in (a)(ii)					

Question			Generic scheme	Illustrative scheme	Max mark
15.	(b)	(i)	•[5] Find resultant velocity •[6] Calculate angle	•[5] $\begin{pmatrix} 832.6 \\ -50 \\ 0 \end{pmatrix}$ •[6] 3.4°	**2**
Notes:					
Commonly Observed Responses:					
		(ii)	•[7] Find displacement vector •[8] Find displacement after 90 mins •[9] Find horizontal component	•[7] $\begin{pmatrix} 832.6t+108 \\ -50t \\ 11.6 \end{pmatrix}$ •[8] $\begin{pmatrix} 1190.37 \\ -65 \\ 11.6 \end{pmatrix}$ •[9] 1192.1	**3**
Notes:					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
16.	(a)		•[1] use conservation of energy with substitution •[2] solve for v	•[1] $20 = \frac{1}{2} \times 0.1 \times v^2 + 0.1 \times 9.8 \times 10$ •[2] 14.3	**2**
Notes:					
Commonly Observed Responses:					
	(b)		•[3] calculate speed •[4] use conservation of energy to calculate height	•[3] $\sqrt{41}\,[6.40]$ •[4] 18.3	**2**
Notes: Evidence for •[3] may appear in working for •[4]					
Commonly Observed Responses:					
	(c)		•[5] use horizontal velocity to calculate kinetic energy	•[5] 0.8	**1**
Notes:					
Commonly Observed Responses:					

[END OF MARKING INSTRUCTIONS]