National
Qualifications
2019

2019 Mathematics of Mechanics

Advanced Higher

Finalised Marking Instructions

SQA

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General marking principles for Advanced Higher Mathematics of Mechanics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example 6 x 6 = 12, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$x = 1$$

This is a transcription error and so the mark is not awarded.

This is no longer a solution of a quadratic equation, so the mark is not awarded.

The following example is an exception to the above

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$(x-3)(x-1) = 0$$
$$x = 1 \text{ or } 3$$

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

(i) **Horizontal/vertical marking**
If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x = 2$	$x = -4$
$\bullet^6$	$y = 5$	$y = -7$

Horizontal: $\bullet^5$ $x = 2$ and $x = -4$
$\bullet^6$ $y = 5$ and $y = -7$

Vertical: $\bullet^5$ $x = 2$ and $y = 5$
$\bullet^6$ $x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\qquad$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0\cdot3}$ must be simplified to 50 $\qquad$ $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $(x^3 + 2x^2 + 3x + 2)(2x + 1)$ written as

  $(x^3 + 2x^2 + 3x + 2) \times 2x + 1$

  $= 2x^4 + 5x^3 + 8x^2 + 7x + 2$

  gains full credit

- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

(q) Any rounded answer should be accurate to three significant figures (or one decimal place for angles given in degrees) unless otherwise stated. If an answer differs due to rounding or prior rounding the candidate may be penalised. Only penalise one mark in any question.

Marking instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.			•¹ use impulse = change in momentum •² calculate final velocity •³ calculate magnitude of velocity •⁴ calculate direction of velocity	•¹ $4\mathbf{v}-4(3\mathbf{i}+2\mathbf{j})=(6\mathbf{i}+\mathbf{j})$ •² $\mathbf{v}=\frac{18\mathbf{i}+9\mathbf{j}}{4}=\frac{9}{2}\mathbf{i}+\frac{9}{4}\mathbf{j}$ •³ $\lvert\mathbf{v}\rvert=\sqrt{\left(\frac{9}{2}\right)^2+\left(\frac{9}{4}\right)^2}=5\cdot03$ •⁴ $\tan^{-1}\left(\frac{9}{4}\div\frac{9}{2}\right)=26\cdot6°$	**4**

Notes:

1. Accept 153·4°

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
2.	(a)		•1 start to use the product rule with one term correct •2 complete differentiation •3 substitute $x=-1$	•1 $1\times e^{-3x}$ ... or ... $-3xe^{-3x}$ •2 $e^{-3x}-3xe^{-3x}$ •3 $4e^{3}$	**3**
Notes:					
Commonly Observed Responses:					
	(b)		•4 start differentiation with evidence of use of quotient rule with denominator and one term of numerator correct •5 complete differentiation •6 simplify answer	•4 $\dfrac{3(2t+1)^2\ldots}{\left((2t+1)^2\right)^2}$ or $\dfrac{\ldots-3t\left(2(2t+1)\times2\right)}{\left((2t+1)^2\right)^2}$ •5 $\dfrac{3(2t+1)^2-3t\left(2(2t+1)\times2\right)}{\left((2t+1)^2\right)^2}$ •6 $\dfrac{3(1-2t)}{(2t+1)^3}$	**3**
Notes: 1. •6 accept $\dfrac{3-6t}{(2t+1)^3}$ 2. •6 is not available for a candidate who produces further incorrect simplification.					
Commonly Observed Responses:					
Alternative solution for (b) – Product rule					
			•4 start differentiation with evidence of use of product rule with one term correct •5 complete differentiation •6 simplify answer	•4 $3(2t+1)^{-2}$ ... or ... $-3t\left(2(2t+1)^{-3}\times2\right)$ •5 $3(2t+1)^{-2}-3t\left(2(2t+1)^{-3}\times2\right)$ •6 $\dfrac{3(1-2t)}{(2t+1)^3}$	

Question			Generic scheme	Illustrative scheme	Max mark
3.			•[1] integrate both components	•[1] $4t+c_1$ and $\frac{t^2}{2}+t+c_2$	4
			•[2] evaluate constant(s) of integration	•[2] $c_1=c_2=0$ as boat starts at origin	
			•[3] calculate displacement after 10 seconds	•[3] $40\mathbf{i}+60\mathbf{j}$	
			•[4] find distance and state if within range.	•[4] $72\cdot1$ Yes, it is within range	

Notes:
If constants of integration are omitted at •[1], award •[1] but •[2] is unavailable

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
4.			•[1] use maximum speed and acceleration in appropriate formulae	•[1] $15=a\omega$ and $60=a\omega^2$	5
			•[2] state values of a and ω	•[2] $\omega=4$ $\quad a=\frac{15}{4}$	
			•[3] derive or state equation for velocity at an instant	•[3] $a\omega\cos\omega t$	
			•[4] substitute to give value of velocity	•[4] $-2\cdot18$	
			•[5] interpret velocity	•[5] particle is moving in opposite direction to original movement	

Notes:
1. •[5] is unavailable for a positive answer at •[4]

Commonly Observed Responses:
Award •[3] for $x=\frac{15}{4}\sin(4\times2)=3\cdot71$ **and** $v^2=4^2\left(\left(\frac{15}{4}\right)^2-3\cdot71^2\right)$
Subsequently, •[4] can only be awarded for selecting the negative value with appropriate justification

Question			Generic scheme	Illustrative scheme	Max mark
5.			•1 state auxiliary equation •2 solve auxiliary equation and state general solution •3 differentiate general solution •4 substitute values into general solution and derivative to obtain 2 equations in A and B •5 solve for A and B and state solution	•1 $m^2-3m+2=0$ •2 $y=Ae^x+Be^{2x}$ •3 $\frac{dy}{dx}=Ae^x+2Be^{2x}$ •4 $1=A+B$ $3=A+2B$ •5 $y=-e^x+2e^{2x}$	**5**
Notes: 1. "$\ldots=0$ " must appear for •1 to be awarded 2. "$y=$ …" need not appear at •2, but must appear in the final answer for •5 to be awarded					
Commonly Observed Responses:					
6.	(a)		•1 take moments about support •2 find magnitude of turning effect •3 interpret answer	•1 $10g\times4+5g\times1-12g\times2$ •2 $45g-24g=21g$ •3 anticlockwise	**3**
	(b)		•4 take moments about any point •5 equate to moments in opposite direction •6 calculate required distance	•4 $10g(4-x)+5g(1-x)$ or $30gx+12g(x+2)$ •5 $10g(4-x)+5g(1-x)=30gx+12g(x+2)$ •6 $\frac{21}{57}$ or $0\cdot368$	**3**
Notes: •2 Accept 206					
Alternative solution for (b)					
			•4 calculate total mass and start to take moments about A •5 complete moments about A •6 calculate required distance	•4 $57x=\ldots$ •5 $57x=5\times3+12\times6+30\times4$ •6 $x=3\cdot632\Rightarrow0\cdot368$	

Question			Generic scheme	Illustrative scheme	Max mark
7.			•¹ begin to differentiate log function •² differentiate either trig term •³ complete differentiation •⁴ simplify	•¹ $\dfrac{1}{(\sec 2t+\tan 2t)}\ldots$ •² $2\sec 2t\tan 2t$ or $2\sec^2 2t$ •³ $\dfrac{2\sec 2t\tan 2t+2\sec^2 2t}{(\sec 2t+\tan 2t)}$ •⁴ $2\sec 2t$	4
Notes: •⁴ accept $\dfrac{2}{\cos 2t}$					
Commonly Observed Responses:					
8.			•¹ set up integral •² begin integration by parts •³ complete integration and include constant of integration •⁴ determine value of c from initial conditions •⁵ determine value of velocity	•¹ $\int 2t(2t+1)^{\frac{1}{2}}\,dt$ •² $2t\times\dfrac{(2t+1)^{\frac{3}{2}}}{\frac{3}{2}\times 2}-\ldots$ •³ $2t\times\dfrac{(2t+1)^{\frac{3}{2}}}{\frac{3}{2}\times 2}-\dfrac{2}{3}\times\dfrac{(2t+1)^{\frac{5}{2}}}{\frac{5}{2}\times 2}+c$ •⁴ $c=\dfrac{2}{15}$ •⁵ $v=39\cdot 7$	5
Notes: 1. Alternative method for •³ •⁴ •⁵ could involve using limits of integration. In this case •⁴ is awarded for correct limits. 2. $\ldots dt$ must appear somewhere in the working for •¹ to be awarded					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
9.			•1 resolve forces parallel to the plane •2 resolve forces perpendicular to the plane •3 use equations from •1 and •2 to eliminate F •4 solve to find θ •5 substitute value for θ into either equation for F and solve	•1 $F\cos\theta + 25 = mg\sin 40$ •2 $F\sin\theta + 30 = mg\cos 40$ •3 $\frac{\sin\theta°}{\cos\theta°} = \frac{5g\cos 40 - 30}{5g\sin 40 - 25}$ •4 $49\cdot2°$ •5 $9\cdot95$	**5**

Notes:
1. For •5 accept $9\cdot94$ or $9\cdot96$

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
10.			•1 start to differentiate using product rule •2 complete differentiation •3 determine value of x when $y = 0$ •4 evaluate gradient	•1 $\ldots 2xe^{2y}\ldots$ or $\ldots 2x^2e^{2y}\frac{dy}{dx}\ldots$ •2 $3\frac{dy}{dx} + 2xe^{2y} + 2x^2e^{2y}\frac{dy}{dx} = 0$ •3 $x = 3$ •4 $-\frac{2}{7}$ or $-0\cdot286$	**4**

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
11.			•[1] use Newton's second law with substitution to set up equation •[2] separate variables and set up integration •[3] integrate with constant of integration (or use of limits) •[4] find constant of integration •[5] substitute and rearrange equation for v	•[1] $-0\cdot2v^2 = 2v\dfrac{dv}{dx}$ •[2] $\int -0\cdot1dx = \int \dfrac{1}{v}dv$ •[3] $-0\cdot1x + c = \ln\lvert v\rvert$ •[4] $c = \ln 5$ •[5] $-0\cdot1x + \ln 5 = \ln\lvert v\rvert$ $v = 5e^{-0\cdot1x}$	**5.**

Notes:

1. If c is omitted at •[3], then •[3], •[4] and •[5] are not available.
2. Do not withhold •[3] or •[5] for the omission of the modulus sign
3. Alternative method for •[3] •[4] •[5] could involve using limits of integration
4. for •[1] accept $-0\cdot2v^2 = 2\dfrac{dv}{dt}$. All marks are still available for appropriate working.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
12.	(a)		•1 resolve forces vertically •2 apply Newton's 2nd law for horizontal forces •3 substitute and eliminate R •4 substitute in expression for v and use trig identity for $\tan\theta°$ •5 rearrange to required answer	•1 $R\cos\theta° + \mu R\sin\theta° = mg$ •2 $R\sin\theta° - \mu R\cos\theta° = \frac{mv^2}{r}$ •3 $\frac{\sin\theta° - \mu\cos\theta°}{\cos\theta° + \mu\sin\theta°} = \frac{v^2}{gr}$ •4 $\frac{\tan\theta° - \mu}{1 + \mu\tan\theta°} = \frac{1}{100}$ •5 $100\tan\theta° - 100\mu = 1 + \mu\tan\theta°$ $\mu\tan\theta° + 100\mu = 100\tan\theta° - 1$ $\mu = \frac{100\tan\theta° - 1}{\tan\theta° + 100}$.	5

Notes:
1. •5 is unavailable for candidates who write down the correct expression without justification

Commonly Observed Responses:

	(b)		•6 resolve forces for friction acting down the slope •7 substitute and eliminate R •8 find maximum speed •9 find minimum speed and state conclusion	•6 $R\cos\theta° - \mu R\sin\theta° = mg$ $R\sin\theta° + \mu R\cos\theta° = \frac{mv^2}{r}$ •7 $\frac{\sin\theta° + \mu\cos\theta°}{\cos\theta° - \mu\sin\theta°} = \frac{v^2}{gr}$ •8 $v = 30 \cdot 3$ •9 $2 \cdot 8$ and motorcyclist will not slip.	4

Notes:
1. Accept •7 stated immediately from (a) as understanding of slipping up the slope

Commonly Observed Responses:

	(c)		•10 state reason with justification	•10 eg worn tyres - alter value of coefficient of friction	1

Notes:
1. •10 cannot be awarded for any reference to mass

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
13.	(a)		•¹ resolve perpendicular to the slope •² apply Newton's second law parallel to the slope •³ find expression for acceleration •⁴ substitute into equation of motion and complete	•¹ $R = mg\cos\theta$ •² $-\mu R - mg\sin\theta = ma$ •³ $a = -g(\mu\cos\theta + \sin\theta)$ •⁴ $0 = V^2 + 2(-g(\mu\cos\theta + \sin\theta))s$ $s = \dfrac{V^2}{2g(\mu\cos\theta + \sin\theta)}$	**4**
Notes:					
Commonly Observed Responses:					
	(b)		•⁵ find work done against friction in terms of given variables •⁶ substitute for W and start simplification •⁷ state expression for μ	•⁵ $W = \mu mg\cos\theta \times \dfrac{V^2}{2g(\mu\cos\theta + \sin\theta)}$ •⁶ $\dfrac{1}{8} = \dfrac{\mu\cos\theta}{2(\mu\cos\theta + \sin\theta)}$ •⁷ $\mu = \dfrac{1}{3}\tan\theta$	**3**
Notes: •⁷ **accept** $\mu = \dfrac{\sin\theta}{3\cos\theta}$					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
Alternative solutions for 13. (a)					
			•1 state force acting down slope •2 find work done against friction to travel *s* metres up slope •3 resolve perpendicular to slope and substitute for *R* •4 use work energy principle to find expression for *s*	•1 $F = mg\sin\theta + \mu R$ •2 $(mg\sin\theta + \mu R)s$ •3 $R = mg\cos\theta$ $(mg\sin\theta + \mu mg\cos\theta)s$ •4 $\frac{1}{2}mV^2 = mg(\sin\theta + \mu\cos\theta)s$ $s = \frac{V^2}{2g(\sin\theta + \mu\cos\theta)}$	
			•1 find work done against gravity •2 find work done against friction •3 use work/energy principle •4 find expression for *s*	•1 $mg \times s\sin\theta$ •2 $\mu mg \times s\cos\theta$ •3 $\frac{1}{2}mV^2 = mgs\sin\theta + \mu mgs\cos\theta$ •4 $s = \frac{V^2}{2g(\sin\theta + \mu\cos\theta)}$	

Question			Generic scheme	Illustrative scheme	Max mark
14.	(a)		•[1] consider energy at A •[2] consider energy at P, and substitute for h •[3] use conservation of energy •[4] substitute and calculate angle	•[1] $E_k + E_p = \frac{1}{2}mu^2 + 0$ •[2] $E_k + E_p = \frac{1}{2}mv^2 + mgh$ $= mgr(1-\cos\theta)$ •[3] $\frac{1}{2}mu^2 = mgr(1-\cos\theta)$ •[4] $6\cdot125 = 3\cdot92(1-\cos\theta)$ $\theta = 124\cdot2^\circ$	**4**

Notes:

1. Accept $\theta = 124^\circ$
2. •[1] and •[2] may be implied by •[3]
3. If •[3] does not appear then evidence for •[1] and •[2] must include $E_k + E_p$ or "energy at A" or similar

Commonly Observed Responses:

	(b)		•[5] state requirements for complete circle •[6] set up inequality with initial kinetic energy greater than final potential energy •[7] solve for u	•[5] $v > 0$ when angle = 180° •[6] $\frac{1}{2}mu^2 > 2mgr$ •[7] $u > \sqrt{\frac{8g}{5}}$	**3**

Notes:

1. •[5] may be implied by •[6]
2. •[5] and •[6] can be awarded for equalities
3. •[7] accept $u > 3\cdot96$
4. •[7] do not accept $u \geq 3\cdot96$ or $u \geq \sqrt{\frac{8g}{5}}$

Commonly Observed Responses:

	(c)		•[8] state assumption	•[8] ball is of the same radius as tubing or does not spin or ball is smooth.	**1**

Notes:

Commonly Observed Responses:

<table>
<tr><th colspan="3">Question</th><th>Generic scheme</th><th>Illustrative scheme</th><th>Max mark</th></tr>
<tr><td>15.</td><td>(a)</td><td></td><td>•1 state condition for maximum height

•2 find vertical component of initial velocity and substitute into vertical equation of motion

•3 introduce inequality and complete proof</td><td>•1 $v=0$ stated or implied by •2

•2 $0=u^2\sin^2\theta-2\times g\times s$

•3 $\sin\theta<\frac{\sqrt{2\times g\times 3}}{u}$
$\sin\theta<\frac{\sqrt{6g}}{u}$</td><td>3</td></tr>
<tr><td colspan="6">Notes:
1. Only accept $\sin\theta=\frac{\sqrt{2gs}}{u}$ leading to inequality if further explanation is given</td></tr>
<tr><td colspan="6">Alternative solution for (a)</td></tr>
<tr><td></td><td></td><td></td><td>•1 state expression for height

•2 state expression for time and start substitution

•3 introduce inequality and complete proof</td><td>•1 $ut\sin\theta-\frac{1}{2}gt^2$

•2 $t=\frac{u\sin\theta}{g}$
$u\left(\frac{u\sin\theta}{g}\right)\sin\theta-\frac{1}{2}g\left(\frac{u\sin\theta}{g}\right)^2$

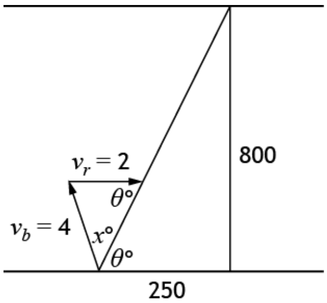
•3 $\ldots<3$ and working leading to
$\sin\theta<\sqrt{\frac{6g}{u}}$</td><td></td></tr>
</table>

Question			Generic scheme	Illustrative scheme	Max mark
15.	(b)	(i)	•4 state time of flight	•4 $\dfrac{2u\sin\theta}{g}$	**5**
			•5 substitute into expression for range	•5 $\dfrac{2u^2\sin\theta\cos\theta}{g}$	
			•6 obtain expression for $\cos\theta$	•6 $\cos\theta=\dfrac{\sqrt{u^2-6g}}{u}$	
			•7 substitute expressions for $\sin\theta$ and $\cos\theta$ into expression for range	•7 $\dfrac{2u^2}{g}\times\dfrac{\sqrt{6g}}{u}\times\dfrac{\sqrt{u^2-6g}}{u}$	
			•8 simplify and complete	•8 valid working leading to $R=12\sqrt{\dfrac{u^2-6g}{6g}}$	
Alternative solution for (b) (i)					
			•4 substitute into 2 equations of motion	•4 $3=\dfrac{(u+v)t}{2}$ $\quad 6=u\sin\theta\times t$	
			•5 combine equations to eliminate $\sin\theta$	•5 $\dfrac{6}{ut}=\dfrac{\sqrt{6g}}{u}$	
			•6 find expression for total time of flight	•6 Total time of flight $=\dfrac{12}{\sqrt{6g}}$	
			•7 find expression for horizontal component of velocity	•7 $u\cos\theta=\sqrt{u^2\left(1-\sin^2\theta\right)}$ $u\cos\theta=\sqrt{u^2-6g}$ $u\cos\theta=\sqrt{u^2-6g}$	
			•8 use expression for range and simplify as required	•8 Range $=\dfrac{12}{\sqrt{6g}}u\cos\theta$ Range $=\dfrac{12\sqrt{u^2-6g}}{\sqrt{6g}}=12\sqrt{\dfrac{u^2-6g}{6g}}$	
		(ii)	•9 state constraint	•9 $u>\sqrt{6g}$	**1**

Notes:

Accept $u\geq\sqrt{6g}$, $u^2\geq 6g$ or $u^2>6g$

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
16.	(a)		•[1] calculate the angle for direct route	•[1] $\tan\theta° = \frac{800}{250}$ $\theta° = 72{\cdot}6°$ 	**4**
			•[2] use sine rule	•[2] $\frac{\sin x°}{2} = \frac{\sin 72{\cdot}6°}{4}$	
			•[3] determine angle inside velocity components triangle	•[3] $x = 28{\cdot}5$	
			•[4] interpret solution	•[4] angle to bank is $101{\cdot}1°$ or $78{\cdot}9°$	

Notes:
•[4] accept $101{\cdot}2°$ or $78{\cdot}8°$

Commonly Observed Responses:

	(b)	(i)	•[5] calculate resultant speed before slowing	•[5] $v_{\text{resultant}} = 4{\cdot}11$	**3**
			•[6] calculate distance from A of rower after 60 seconds	•[6] 247	
			•[7] calculate remaining distance after slowing	•[7] 591	

Alternative solution for (b) (i)

			•[5] set up distance triangle and use sine/cosine rule	•[5] $\frac{x}{\sin 78{\cdot}8} = \frac{120}{\sin 28{\cdot}5} = \frac{240}{\sin 72{\cdot}6}$ or $x^2 = 120^2 + 240^2 - 2\times120\times240\times\cos 78{\cdot}8$	
			•[6] calculate full or partial distance	•[6] 247 after 1 minute or 838 to B	
			•[7] calculate remaining distance	•[7] 591	

Notes:
Accept 592 for •[7]

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
16.	(b)	(ii)	•[8] calculate the new angle with the river bank or angle marked x •[9] calculate resultant velocity after slowing •[10] calculate remaining and total times	•[8] $67\cdot8°$ or $112\cdot2°$ or 39·5 •[9] $v = 2\cdot91\,\text{ms}^{-1}$ •[10] t = 203 seconds, Total time = 263 seconds	**3**
Notes:					
Commonly Observed Responses:					
17.	(a)		•[1] recognise form of integral and integrate correctly	•[1] $\tan\left(e^t\right)+c$	**1**
Notes: constant of integration not required					
Commonly Observed Responses:					
	(b)		•[2] recognise expression for velocity •[3] explain why original function cannot ever equal zero	•[2] $v = e^t \sec^2\left(e^t\right)$ •[3] neither $\sec\left(e^t\right)$ nor e^t can ever equal zero, so product can never be zero and hence particle never at rest	**2**
Notes:					
Commonly Observed Responses:					

[END OF MARKING INSTRUCTIONS]