National
Qualifications
2018

2018 Mathematics of Mechanics

Advanced Higher

Finalised Marking Instructions

General marking principles for Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example 6 x 6 = 12, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.

This is no longer a solution of a quadratic equation, so the mark is not awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$x = 1$$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$(x-3)(x-1) = 0$$
$$x = 1 \text{ or } 3$$

(i) **Horizontal/vertical marking**
If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x = 2$	$x = -4$
$\bullet^6$	$y = 5$	$y = -7$

Horizontal: $\bullet^5$ $x = 2$ and $x = -4$
$\bullet^6$ $y = 5$ and $y = -7$

Vertical: $\bullet^5$ $x = 2$ and $y = 5$
$\bullet^6$ $x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$

$\frac{43}{1}$ must be simplified to 43

$\frac{15}{0\cdot3}$ must be simplified to 50

$\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $(x^3 + 2x^2 + 3x + 2)(2x + 1)$ written as

  $(x^3 + 2x^2 + 3x + 2) \times 2x + 1$

  $= 2x^4 + 5x^3 + 8x^2 + 7x + 2$

  gains full credit

- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

(q) Any rounded answer should be accurate to three significant figures (or one decimal place for angles given in degrees) unless otherwise stated. If an answer differs due to rounding or prior rounding the candidate may be penalised. Only penalise one mark in any question.

Detailed marking instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.			•[1] choose appropriate equation of motion and substitute to find acceleration.	•[1] $v^2 = u^2 + 2as$ $14^2 = 10^2 + 2 \times 1200 \times a$ $a = \frac{14^2 - 10^2}{2400} = \frac{96}{2400} = 0 \cdot 04\,\text{ms}^{-2}$	5
			•[2] find final velocity before deceleration.	•[2] $v = u + at = 14 + 0 \cdot 04 \times 120$ $= 18 \cdot 8\,\text{ms}^{-1}$	
			•[3] substitution to find further distance travelled.	•[3] $s = ut + \frac{1}{2}at^2$ $14 \times 120 + \frac{1}{2} \times 0 \cdot 04 \times 120^2 = 1968\,\text{m}$	
			•[4] find stopping distance	•[4] $v^2 = u^2 + 2as$ $0 = 18 \cdot 8^2 - 2 \times 0 \cdot 04 \times s$ $s = 4418$	
			•[5] calculate total distance	•[5] $\text{total} = 1200 + 1968 + 4418$ $= 7586\,\text{m}$ $[7 \cdot 59\,\text{km}]$	

Notes:
1. accept distance answers in metres or kilometres

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
2.	(a)		•1 use correct form of partial fractions •2 equate numerators •3 find one constant •4 find remaining constants and state the partial fractions	$\frac{13+6x+5x^2}{(1+x)(2-x)(3+x)}$ •1 $=\frac{A}{1+x}+\frac{B}{2-x}+\frac{C}{3+x}$ •2 $13+6x+5x^2=A(2-x)(3+x)+B(1+x)(3+x)+C(2-x)(1+x)$ •3 $A=2$ or $B=3$ or $C=-4$ •4 $A=2,\ B=3,\ C=-4$ $\frac{2}{1+x}+\frac{3}{2-x}-\frac{4}{3+x}$	**4**
Notes:					
Commonly Observed Responses:					
	(b)		•5 rewrite integral and integrate one term correctly •6 complete integration •7 substitute and simplify to correct form	•5 $\int_0^1 \frac{2}{1+x}+\frac{3}{2-x}-\frac{4}{3+x}\,dx$ $=2\ln\lvert 1+x\rvert ....$ •6 $....-3\ln\lvert 2-x\rvert-4\ln\lvert 3+x\rvert$ •7 $(2\ln 2-3\ln 1-4\ln 4)$ $-(2\ln 1-3\ln 2-4\ln 3)$ $=\ln\frac{81}{8}$	**3**
Notes:					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
3.			•[1] use Newton's second law with frictional force •[2] calculate the deceleration •[3] calculate speed immediately before the collision •[4] know to use conservation of momentum and start substitution •[5] calculate v	•[1] $ma = -\mu R$ $ma = -\mu mg$ •[2] $a = -\mu g$ $a = -2\cdot 45\,\text{ms}^{-2}$ $\left[\frac{-g}{4}\text{ms}^{-2}\right]$ •[3] $v^2 = 12^2 + 2\times -2\cdot 45\times 20$ $v = 6\cdot 78\,\text{ms}^{-1}$ •[4] $m_1u_1 + m_2u_2 = (m_1 + m_2)v$ •[5] $10 + 6\cdot 78 + 5\times 0 = 15v$ $v = 4\cdot 52\,\text{ms}^{-1}$ $\left[\frac{2\sqrt{46}}{3}\right]$	5

Notes:

1. •[4] initial or final momentum should begin to be calculated

Commonly Observed Responses:

•[3] $a = +2\cdot 45\,\text{ms}^{-2}$ leading to •[5] $v = 10\cdot 4\,\text{ms}^{-1}$

Question			Generic scheme	Illustrative scheme	Max mark
Alternative solution (work/energy principle)					
3.			•[1] consider energy at start and immediately before collision	•[1] start $E_K = \frac{1}{2}mv^2 = \frac{1}{2}\times 10\times 12^2 = 720J$ before collision $E_K = \frac{1}{2}mv^2$	5
			•[2] calculate work done by friction	•[2] Friction $= \mu mg = 0\cdot 25\times 10\times 9\cdot 8 = 24\cdot 5N$ work $= Fs = 24\cdot 5\times 20 = 490J$	
			•[3] use conservation of energy to calculate speed just before collision	•[3] $720 - \frac{1}{2}mv^2 = 490 \Rightarrow \frac{1}{2}\times 10\times v^2 = 230J$ $v = \sqrt{\frac{230\times 2}{10}} = \sqrt{46} = 6\cdot 78ms^{-1}$	
			•[4] know to use conservation of momentum and start substitution	•[4] $m_1u_1 + m_2u_2 = (m_1 + m_2)v$	
			•[5] calculate v	•[5] $10 + 6{\cdot}78 + 5 \times 0 = 15v$ $v = 4\cdot 52ms^{-1}$	

Notes:

1. •[4] initial or final momentum should begin to be calculated

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
4.			•1 start to use chain rule to find derivative •2 complete the differentiation •3 substitute $x = \frac{\pi}{4}$	•1 $f\,'(x) = e^{\sec^2 x} \times \frac{d}{dx}\sec^2 x$ •2 $2\sec^2 x \tan x\, e^{\sec^2 x}$ •3 $\sec\frac{\pi}{4} = \sqrt{2}$ $\quad \sec^2\frac{\pi}{4} = 2$ $\quad \tan\frac{\pi}{4} = 1$ $f\,'\left(\frac{\pi}{4}\right) = 2\times 2\times 1\times e^2$	3

Notes:

1. •1 clear evidence to show multiplication by the ***derivative*** of $\sec^2 x$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
5.			•¹ denotes quantities appropriately (via diagram or otherwise) and resolve vertically •² use Newton's 2nd law horizontally with circular motion •³ eliminate T and m •⁴ use $l = 2r$ to find a value for $\tan\theta$ or evaluate θ •⁵ complete proof	•¹ $T\cos\theta = mg$ •² $T\sin\theta = mr\omega^2$ •³ $\tan\theta = \frac{r\omega^2}{g}$ •⁴ $\tan\theta = \frac{1}{\sqrt{3}}$ $\left[\sin\theta = \frac{1}{2} \quad \theta = 30^\circ\right]$ $\frac{1}{\sqrt{3}} = \frac{l\omega^2}{2g}$ •⁵ $\sqrt{3}l\omega^2 = 2g$ $\omega^2 = \frac{2g}{\sqrt{3}l}$	**5**
Notes:					
Commonly Observed Responses:					

Question			Generic Scheme	Illustrative Scheme	Max Mark
6.			•1 express volume as an integral	•1 $V=\pi\int y^2dx$	4
			•2 use integral with limits substitute for y^2	•2 $V=\pi\int_{-2}^{3}\left(9-x^2\right)dx$	
			•3 integrate	•3 $V=\pi\left[9x-\frac{1}{3}x^3\right]_{-2}^{3}$	
			•4 evaluate	•4 $\frac{100\pi}{3}$ [105]	

Notes:

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
7.	(a)		•[1] calculate ω	•[1] $\omega=\frac{2\pi}{10}$ $\omega=\frac{\pi}{5}$	4
			•[2] state equation for position and start to solve	•[2] $x=6\sin\frac{\pi}{5}t$ $6\sin\frac{\pi}{5}t=4$	
			•[3+4] obtain values for t	•[3+4] $\frac{\pi}{5}t=0\cdot730,\ 2\cdot41$ $t=1\cdot16,\ 3\cdot84$	

Notes:
1. •[3+4] Horizontal and vertical marking.

Commonly Observed Responses:

	(b)		**Method 1**	**Method 1**	2
			•[5] use second value of t to find v	•[5] $v=a\omega\cos\omega t$ $v=\frac{6\pi}{5}\cos(\frac{\pi}{5}\times3\cdot84)$	
			•[6] evaluate and interpret solution	•[6] $v=-2{\cdot}81\,\text{ms}^{-1}$ so particle will be travelling back towards A with speed of $2{\cdot}81\text{ms}^{-1}$	

Notes:
•[6] only available where v is negative.

Commonly Observed Responses:

	(b)		**Method 2**	**Method 2**	2
			•[5] use second value of t to find v	•[5] $v^2=\omega^2(a^2-x^2)$ $v^2=\left(\frac{\pi}{5}\right)^2\left(6^2-4^2\right)$	
			•[6] evaluate and interpret solution.	•[6] $v=-2{\cdot}81\,\text{ms}^{-1}$ so for second time particle will be travelling back towards A with a speed of $2{\cdot}81\,\text{ms}^{-1}$.	

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
8.			•[1] find $\frac{dx}{dt}$ •[2] find $\frac{dy}{dt}$ •[3] evaluate derivatives when $t = 3$ •[4] substitute into appropriate formula and calculate speed	•[1] $\frac{dx}{dt} = 2t + 4$ •[2] $\frac{dy}{dt} = (1-t)^3 - 3t(1-t)^2$ •[3] $\frac{dx}{dt}(t = 3) = 10$ and $\frac{dy}{dt}(t = 3) = -44$ •[4] $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$ $\sqrt{10^2 + (-44)^2} = \sqrt{2036} = 45 \cdot 1$	**4**

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
9.	(a)		**Method 1**	**Method 1**	4
			•1 use appropriate formula for time of half flight with substitution	•1 $v=u+at \Rightarrow 0=v\sin\theta-gt$	
			•2 find expression for total time of flight	•2 $t=\dfrac{v\sin\theta}{g} \Rightarrow 2t=\dfrac{2v\sin\theta}{g}$	
			•3 find expression for range using total time of flight	•3 $R=v\cos\theta\times 2t=\dfrac{v\cos\theta\times 2v\sin\theta}{g}$	
			•4 simplify using double angle formula	•4 $R=\dfrac{v^2\times 2\sin\theta\cos\theta}{g}=\dfrac{v^2\sin 2\theta}{g}$	
Notes:					
Commonly Observed Responses:					
	(a)		**Method 2**	**Method 2**	4
			•1 state horizontal range of flight and use it to give expression for t	•1 $R=v\cos\theta\times t$	
			•2 use appropriate formula with substitution	•2 $s=ut+\dfrac{1}{2}at^2$ $0=v\sin\theta\times t-\dfrac{1}{2}gt^2$	
			•3 solve the equation for t	•3 $0=t\left(v\sin\theta-\dfrac{1}{2}gt\right)$ $[t=0]$ or $t=\dfrac{2v\sin\theta}{g}$	
			•4 substitute for t to give required formula	•4 $R=\dfrac{v\cos\theta\times 2v\sin\theta}{g}=\dfrac{v^2\sin 2\theta}{g}$	
Notes: •3 Do not penalise omission of $t=0$					
Commonly Observed Responses:					

<table>
<tr><th colspan="3">Question</th><th>Generic scheme</th><th>Illustrative scheme</th><th>Max mark</th></tr>
<tr><td>9.</td><td>(a)</td><td></td><td>Method 3

•[1] consider horizontal and vertical motion

•[2] set up equation for vertical motion at start and finish

•[3] solve the equation for t

•[4] substitute for end value for t to give range formula</td><td>Method 3

•[1] $\ddot{x}=0 \Rightarrow \dot{x}=v\cos\theta \Rightarrow x=vt\cos\theta$
$\ddot{y}=-g \Rightarrow \dot{y}=-gt+v\sin\theta$
$\Rightarrow y=-\frac{1}{2}gt^2+vt\sin\theta$

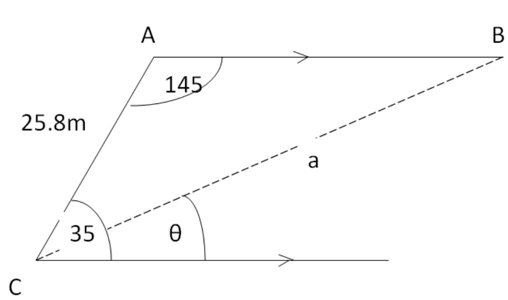
•[2] $y=-\frac{1}{2}gt^2+vt\sin\theta=0$

•[3] $0=t\left(v\sin\theta-\frac{1}{2}gt\right)$
$[t=0]$ or $t=\dfrac{2v\sin\theta}{g}$

•[4] $x=v\cos\theta\times t=\dfrac{v\cos\theta\times 2v\sin\theta}{g}$
$=\dfrac{v^2\sin 2\theta}{g}$</td><td>4</td></tr>
<tr><td colspan="6">Notes:
•[3] Do not penalise omission of $t=0$</td></tr>
<tr><td colspan="6">Commonly Observed Responses:</td></tr>
</table>

Question			Generic scheme	Illustrative scheme	Max mark
9.	**(b)**	**(i)**	•[5] substitute both angles into range formula •[6] by substituting for R set up equation in v •[7] re-arrange and solve for v	•[5] $R = \frac{v^2 \sin 60°}{g}$ $R + 5 = \frac{v^2 \sin 70°}{g}$ •[6] $\frac{v^2 \sin 60°}{g} + 5 = \frac{v^2 \sin 70°}{g}$ •[7] $\frac{v^2(\sin 70° - \sin 60°)}{g} = 5$ $v^2 = \frac{5g}{\sin 70° - \sin 60°}$ $[665 \cdot 2]$ $v = 25 \cdot 8 \text{ms}^{-1}$	**3**
Notes:					
Commonly Observed Responses: •[7] not available where calculator set in radians					
		(ii)	•[8] calculate initial velocity when $\theta = 35°$ •[9] calculate time of flight •[10] calculate range with $\theta = 35°$	•[8] $\mathbf{v} = \begin{pmatrix} 25 \cdot 8 \cos 35° + 7 \\ 25 \cdot 8 \sin 35° \end{pmatrix}$ $\left[\begin{pmatrix} 28 \cdot 13 \\ 14 \cdot 80 \end{pmatrix}\right]$ $v = u + at$ $0 = 14 \cdot 8 - gt$ •[9] $t = \frac{14 \cdot 8}{g} = 1 \cdot 51$ Total time $= 3 \cdot 02$ •[10] $28 \cdot 13 \times 3 \cdot 02 = 85 \cdot 0$ metres	**3**
Notes: **1. •[8]** can be implied in further working and does not have to be explicitly stated 2. •[10] accept 85 m or $84 \cdot 9$ m (exact values used throughout)					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
Alternative solution 1					
	(b)	(ii)	•[8] substitute original velocity into range formula for $\theta = 35^{\circ}$ •[9] calculate time of flight •[10] add on extra distance for wind assistance	•[8] $R = \frac{25\cdot8^2 \times \sin 70^{\circ}}{9\cdot8} = 63\cdot8\text{m}$ •[9] $t = \frac{2v\sin\theta}{g} = \frac{2\times25\cdot8\times\sin 35}{9{\cdot}8} = 0\cdot302$ •[10] $R = 63\cdot8 + 7\times0\cdot302 = 84\cdot9\text{m}$	3
Notes:					
Commonly Observed Responses:					
Alternative solution 2					
		(ii)	•[8] find new horizontal component •[9] calculate time of flight •[10] calculate range	•[8] $\ddot{x} = 0 \Rightarrow \dot{x} = v\cos\theta + 7$ $\Rightarrow x = vt\cos\theta + 7t = 21{\cdot}13$ •[9] $t = \frac{2v\sin\theta}{g} = \frac{2\times25\cdot8\times\sin 35}{9{\cdot}8} = 0\cdot302$ •[10] $R = 21{\cdot}13\times3\cdot02 + 7\times3\cdot02 = 85\cdot0\text{m}$	3
Notes:					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
Alternative solution 3					
9.	(b)	(ii)			**3**
			•[8] calculate resultant velocity	$a^2 = b^2 + c^2 - 2bc\cos A$ •[8] $= 25\cdot8^2 + 7^2 - 2(25\cdot8)(7)\cos145°$ $= 1010\cdot5$ $a = 31\cdot8\text{ms}^{-1}$	
			•[9] calculate angle	•[9] $\frac{\sin145°}{31\cdot8} = \frac{\sin C}{7} \Rightarrow C = \sin^{-1}(0\cdot126) = 7\cdot25°$ $\theta = 35° - 7\cdot25° = 27\cdot7°$	
			•[10] calculate range using formula	•[10] $R = \frac{v^2\sin2\theta}{g} = \frac{31\cdot8^2\sin(2\times27\cdot7)}{9\cdot8} = 85\cdot0\text{m}$	
Notes:					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
10.	(a)		**Method1: Relative to A**		**6**
			•[1] derive expressions for the mass and centres of mass of the original lamina and the circular hole	•[1] Original Lamina: $16\pi m$ $(4,0)$ Circular hole: πm $(2,1)$	
			•[2] derive expressions for the mass and centres of mass of the semi-circular hole	•[2] Semi-circular hole: $2\pi m$ $\left(6, \frac{8}{3\pi}\right)$ $[6, 0\cdot849]$	
			•[3] take moments horizontally by equating with centre of mass of remaining shape	•[3] $13\pi m\bar{x} = 16\pi m \times 4 - \pi m \times 2 - 2\pi m \times 6$	
			•[4] solve this equation to find horizontal value of centre of mass	•[4] $\bar{x} = \frac{50}{13}$ $[3\cdot846]$	
			•[5] take moments vertically	•[5] $13\pi m\bar{y} = 16\pi m(0) - \pi m \times 1 - 2\pi m \times \frac{8}{3\pi}$	
			•[6] solve this equation to find vertical value of centre of mass	•[6] $\bar{y} = -0\cdot208$	

Notes:

1. •[6] Position does not have to be specified as coordinates as moments were taken from *A*
2. Do not penalise omission of mass

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
10.	(a)		**Method 2: Relative to C**		**6**
			•[1] derive expressions for the mass and centres of mass of the original lamina and the circular hole	•[1] Original Lamina: $16\pi m$ $(4,0)$ Circular hole: πm $(2,1)$	
			•[2] derive expressions for the mass and centres of mass of the semi-circular hole	•[2] Semi-circular hole: $2\pi m$ $\left(6, \dfrac{8}{3\pi}\right)$ $[6, 0\cdot849]$	
			•[3] take moments horizontally by equating with centre of mass of remaining shape	•[3] $13\pi m\bar{x} = 16\pi m \times 0 - \pi m \times -2 - 2\pi m \times 2$	
			•[4] solve this equation to find horizontal value of centre of mass	•[4] $\bar{x} = \dfrac{-2}{13}$ $[-0\cdot154]$	
			•[5] take moments vertically	•[5] $13\pi m\bar{y} = 16\pi m(0) - \pi m \times 1 - 2\pi m \times \dfrac{8}{3\pi}$	
			•[6] solve this equation to find vertical value of centre of mass And state coordinates relative to *A*	•[6] $\bar{y} = -0\cdot208$ $(3\cdot846, -0\cdot208)$	

Notes:

Commonly Observed Responses:

•[1] •[2] Alternative presentation of data

	Original $\pi m(4^2) = 16\pi m$	Small Circle $\pi m(4^2) = 16\pi m$	Semicircle $\frac{1}{2}\pi m(2^2) = 2\pi$	Remaining $13\pi m$
Moments from A: $\bar{x}$	$\begin{pmatrix}4\\0\end{pmatrix}$	$\begin{pmatrix}2\\1\end{pmatrix}$	$\begin{pmatrix}6\\\frac{8}{3\pi}\end{pmatrix}$	$\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$
Moments from C: $\bar{y}$	$\begin{pmatrix}0\\0\end{pmatrix}$	$\begin{pmatrix}-2\\1\end{pmatrix}$	$\begin{pmatrix}2\\\frac{8}{3\pi}\end{pmatrix}$	$\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$

	(b)		•[7] interpret rotation	•[7] $\tan\theta = \dfrac{0\cdot208}{3\cdot846}$ $\theta = 3\cdot1°$	**1**

Notes:

Commonly Observed Responses:

<table>
<tr><th colspan="3">Question</th><th>Generic scheme</th><th>Illustrative scheme</th><th>Max mark</th></tr>
<tr><td>11.</td><td>(a)</td><td></td><td>•¹ calculate the displacement of A and B in 6 minutes

•² calculate velocity of A and B</td><td>•¹ $\mathbf{r}_A = 4\cdot8\mathbf{i} + 1\cdot4\mathbf{j}$
$\mathbf{r}_B = -0\cdot8\mathbf{i} + 1\cdot5\mathbf{j}$

•² $\mathbf{v}_A = \frac{4\cdot8}{0\cdot1}\mathbf{i} + \frac{1\cdot4}{0\cdot1}\mathbf{j} = 48\mathbf{i} + 14\mathbf{j}$
$\mathbf{v}_B = \frac{-0\cdot8}{0\cdot1}\mathbf{i} + \frac{1\cdot5}{0\cdot1}\mathbf{j} = -8\mathbf{i} + 15\mathbf{j}$</td><td>2</td></tr>
<tr><td></td><td>(b)</td><td>(i)</td><td>•³ express displacement of A and B as functions of time

•⁴ equate $\mathbf{i}$-components

•⁵ equate $\mathbf{j}$-components and form conclusion</td><td>•³ $\mathbf{r}_A = (1\cdot2 + 48t)\mathbf{i} + (1\cdot6 + 14t)\mathbf{j}$
$\mathbf{r}_B = (34\cdot8 - 8t)\mathbf{i} + (1 + 15t)\mathbf{j}$

•⁴ $1\cdot2 + 48t = 34\cdot8 - 8t$
$\mathbf{i}$ components equal when
$t = 0\cdot6$ hours

•⁵ $1\cdot6 + 14t = 1 + 15t$
$t = 0\cdot6$ hours
$\mathbf{i}$ and $\mathbf{j}$ components are equal at
$t = 0\cdot6$ so boats collide</td><td>3</td></tr>
<tr><td colspan="6">Notes:
1. Horizontal marking can apply at •⁴ and •⁵.</td></tr>
<tr><td colspan="6">Commonly Observed Responses:</td></tr>
<tr><td></td><td></td><td>(ii)</td><td>•⁶ find the position of collision</td><td>•⁶ $\begin{pmatrix}30\\10\end{pmatrix}$ or $(30,10)$</td><td>1</td></tr>
<tr><td colspan="6">Notes:</td></tr>
<tr><td colspan="6">Commonly Observed Responses:</td></tr>
</table>

Question			Generic scheme	Illustrative scheme	Max mark
Alternative Solution (relative position vector)					
	(b)	(i)	•³ express displacement of A and B as functions of time •⁴ find relative position vector and set vector or either component to zero •⁵ find time of collision and form conclusion	•³ $\mathbf{r}_A = \begin{pmatrix} 48t+1\cdot2 \\ 14t+1\cdot6 \end{pmatrix}, \mathbf{r}_B = \begin{pmatrix} -8t+34\cdot8 \\ 15t+1 \end{pmatrix}$ •⁴ ${}_A\mathbf{r}_B = \begin{pmatrix} 56t-33\cdot6 \\ -t+0\cdot6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ or $56t-33\cdot6=0$ or $-t+0\cdot6=0$ •⁵ $-t+0\cdot6=0 \Rightarrow t=0\cdot6$ $56t-33\cdot6=0 \Rightarrow t=0\cdot6$ $\mathbf{i}$ and $\mathbf{j}$ components are equal at $t=0\cdot6$ so boats collide	3
		(ii)	•⁶ find the position of collision	•⁶ $\begin{pmatrix} 30 \\ 10 \end{pmatrix}$ or $(30,10)$	1
Notes:					
Commonly Observed Responses:					
Alternative solution (parallel vectors)					
	(b)	(i)	•³ expression to indicate method of bringing *B* to rest with substitution •⁴ expression for A_1B_1 •⁵ *A* and *B* will collide if v_{A-B} is parallel to A_1B_1	$v_{A-B} = v_A - v_B$ •³ $= \begin{pmatrix} 48 \\ 14 \end{pmatrix} - \begin{pmatrix} -8 \\ 15 \end{pmatrix} = \begin{pmatrix} 56 \\ -1 \end{pmatrix}$ •⁴ $A_1B_1 = \begin{pmatrix} 34\cdot8 \\ 1 \end{pmatrix} - \begin{pmatrix} 1\cdot2 \\ 1\cdot6 \end{pmatrix} = \begin{pmatrix} 33\cdot6 \\ -0\cdot6 \end{pmatrix}$ •⁵ $\frac{3}{5}\begin{pmatrix} 56 \\ -1 \end{pmatrix} = \begin{pmatrix} 33\cdot6 \\ -0\cdot6 \end{pmatrix}$ or $A_1B_1 = 0\cdot6v_{A-B}$ So boats collide	3
		(ii)	•⁶ use $t=0\cdot6$ to find the position of collision and state as coordinate	•⁶ $\begin{pmatrix} 30 \\ 10 \end{pmatrix}$ or $(30,10)$	1
Notes:					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
12.	(a)		•¹ use Newton's second law parallel to wire •² resolve perpendicular to the cable and combine equations and simplify expression for acceleration •³ use appropriate equation of motion with some substitution •⁴ substitute all values and calculate speed	•¹ $mg\sin\theta - \mu R = ma$ •² $R = mg\cos\theta$ $a = g\left(\sin\theta - \mu\cos\theta\right)$ $[0\cdot589]$ •³ $v^2 = u^2 + 2\left(g\left(\sin\theta - \mu\cos\theta\right)\right)s$ •⁴ $v^2 = 2^2 + 2\left(g\left(\sin 20° - 0\cdot3\cos 20°\right)\right)\times 20$ $v = 5\cdot25\,\text{ms}^{-1}$	**4**

Notes:

Commonly Observed Responses:

Alternative solution (work/energy principle)

	(a)		•¹ calculate height and find expression for energy at top •² find expression for energy at bottom and calculate change in energy •³ calculate work done against friction and use work/energy principle •⁴ substitute and solve to find speed	$h = 20\sin 20°\left(\approx 6\cdot84\right)$ •¹ and $mg\times 20\sin 20° + \frac{1}{2}m\times 2^2$ •² $20mg\sin 20° + 2m - \frac{1}{2}mv^2$ $W = 0\cdot3mg\cos 20°\times 20$ •³ $W = 20mg\sin 20° + 2m - \frac{1}{2}mv^2$ $6mg\cos 20°$ •⁴ $= 20mg\sin 20° + 2m - \frac{1}{2}mv^2$ $v = 5\cdot25\,\text{ms}^{-1}$	**4**

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
12.	(b)		•[5] find total initial energy •[6] find total final energy •[7] use conservation of energy to form equation •[8] substitute values and calculate angle	•[5] setting zero PE level at seat $E_K + E_P = \frac{1}{2}mu^2 + 0 = 13\cdot 8m$ •[6] $E_K + E_P = 0 + mg(r - r\cos\theta)$ •[7] $13\cdot 8m = mgr(1-\cos\theta)$ •[8] $\cos\theta = 1 - \dfrac{5\cdot 25^2}{2\times 9\cdot 8\times 1\cdot 8}$ $\theta = 77\cdot 4°$	**4**
Notes:					
Commonly Observed Responses:					
Alternative solution (work/energy principle)					
	(b)		•[1] use conservation of energy •[2] substitute to find height •[3] find vertical distance below centre of rotation •[4] calculate angle	•[1] $\frac{1}{2}mv^2 = mgh$ •[2] $h = \dfrac{5\cdot 25^2}{2\times 9\cdot 8} = 1\cdot 406$ •[3] $1\cdot 8 - 1\cdot 406 = 0\cdot 394$ •[4] $\cos^{-1}\left(\dfrac{0\cdot 394}{1\cdot 8}\right) = 77\cdot 4°$	**4**
Notes:					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
13.			•¹ differentiate u with respect to x	•¹ $\frac{du}{dx}=2x$	**6**
			•² evaluate new limits	•² $x=0 \Rightarrow u=4,\ x=\sqrt{5} \Rightarrow u=9$	
			•³ find new integral	•³ $\int_4^9 \frac{u-4}{u^{\frac{1}{2}}}du$	
			•⁴ express in integrable form	•⁴ $\int_4^9 \left(u^{\frac{1}{2}}-4u^{-\frac{1}{2}}\right)du$	
			•⁵ integrate	•⁵ $\left[\frac{2}{3}u^{\frac{3}{2}}-8u^{\frac{1}{2}}\right]_4^9$	
			•⁶ evaluate	•⁶ $\frac{14}{3}$	

Notes:

1. •⁵ only $\frac{14}{3}$ or $4\frac{2}{3}$ are acceptable since the exact value is requested.
2. •² can be awarded for resubstituting for x instead of evaluating new limits.

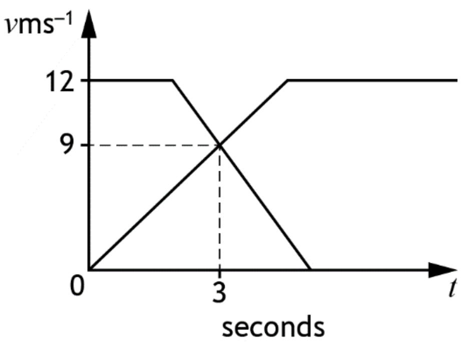
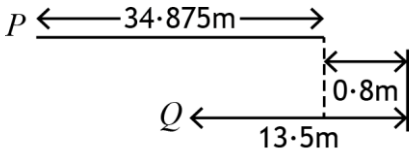
Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
14.			•¹ model EPE in stretched rope	•¹ $\text{EPE} = \frac{1}{2}\frac{\lambda x^2}{l} = 50d^2$	5
			•² equate potential and elastic potential energy at lowest point	•² $E_p = mg(10+d)$ $= 70\times 9.8\times(10+d) = 50d^2$	
			•³ set up quadratic equation in d	•³ $6860 + 686d = \frac{1}{2}\times\frac{1000}{10}d^2$ $50d^2 - 686d - 6860 = 0$	
			•⁴ solve for d	•⁴ $d = \frac{686 \pm \sqrt{686^2 + 4\times 50\times 6860}}{2\times 50}$ $= 20{\cdot}43\ldots$ or $-6{\cdot}71\ldots$	
			•⁵ select appropriate solution and find height above water	•⁵ total length = $10 + 20{\cdot}43 = 30{\cdot}43$ height above water $40 - 30{\cdot}43 = 9{\cdot}57\,\text{m}$	
Alternative for •¹					
			•¹ calculate work done to stretch d	•¹ $W = \int_0^d F dx = \int_0^d (\frac{\lambda}{10}x) dx = \frac{1}{2}\frac{\lambda}{10}d^2$	
Notes:					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
Alternative solution (SHM)					
14.			•[1] calculate speed at point cord becomes tense	$v^2 = u^2 + 2as$ •[1] $\Rightarrow v^2 = 0^2 + 2\times 9\cdot 8\times 10$ $v = 14$	**5**
			•[2] calculate equilibrium extension	•[2] $\frac{\lambda x_e}{l} = mg \Rightarrow \frac{1000x_e}{10} = 70g$ $x_e = 0\cdot 7g = 6\cdot 86$	
			•[3] use Newton's second law to set up equation and calculate ω	•[3] $70g - \frac{1000(x+0\cdot 7g)}{10} = 70\ddot{x}$ $\ddot{x} = -\frac{10}{7}x \Rightarrow \omega = \sqrt{\frac{10}{7}}$	
			•[4] calculate amplitude of motion	•[4] $14^2 = \left(\sqrt{\frac{10}{7}}\right)^2 \left(a^2 - (0\cdot 7g)^2\right)$ $a = 13\cdot 574$	
			•[5] calculate height above water	•[5] $40 - (10 + 6\cdot 86 + 13\cdot 574)$ $= 9\cdot 57\,\text{m}$	
Notes:					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
Alternative solution (Newton's Second Law and splitting the variables)					
14.			•1 apply Newton's Second Law and Hooke's Law •2 separate variables and integrate •3 calculate speed at point cord becomes tense and substitute to find constant of integration •4 substitute $v=0$ and solve quadratic •5 select solution and calculate height above water	•1 $mg-\frac{\lambda x}{l}=ma$ $70g-\frac{1000x}{10}=70v\frac{dv}{dx}$ •2 $\int v\,dv=\int\left(g-\frac{10}{7}x\right)dx$ $\frac{v^2}{2}+c=gx-\frac{5}{7}x^2$ •3 $v^2=0^2+2\times 9\cdot 8\times 10\Rightarrow v=14$ $x=0, v=14\Rightarrow c=-98$ $\therefore \frac{v^2}{2}-98=gx-\frac{5}{7}x^2$ •4 $5x^2-7gx-686=0$ $\ldots\Rightarrow x=20\cdot 43,\ x=-6\cdot 71$ •5 $40-10-20\cdot 43$ $=9\cdot 57\,\text{m}$	**5**
Notes:					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
15.	(a)		•[1] set up auxiliary equation •[2] solve quadratic equation to give general solution •[3] initial condition $x = 1\cdot5$ when $t = 0$ •[4] differentiate to use initial condition •[5] substitution to obtain B and particular solution	•[1] $m^2 + 0\cdot4m + 0\cdot04 = 0$ •[2] $(m + 0\cdot2)(m + 0\cdot2) = 0 \Rightarrow m =$ $-0\cdot2$ repeated $x = Ae^{-0\cdot2t} + Bte^{-0\cdot2t}$ •[3] $A = 1\cdot5$ •[4] $\frac{dx}{dt} = -0\cdot2Ae^{-0\cdot2t} + Be^{-0.2t} - 0\cdot2Bte^{-0\cdot2t}$ •[5] $-0\cdot5 = -0\cdot3 + B$ $B = -0\cdot2$ Hence $x = 1\cdot5e^{-0\cdot2t} - 0\cdot2te^{-0\cdot2t}$	**5**
Notes: 1. •[1] only available for correct quadratic expression equated to zero. 2. •[2] only available if the general solution is expressed in terms of t					
Commonly Observed Responses: •[2] $x = Ae^{-0\cdot2t} + Be^{-0\cdot2t}$, leading to $A + B = 1\cdot5$ only •[1] and •[3] are available. •[5] $\frac{dx}{dt} = +0\cdot5$ leading to $B = 0\cdot8$					
	(b)		•[6] substitute $t = 2$ into expression for x and calculate distance moved.	•[6] $x = 1\cdot5e^{-0\cdot4} - 0\cdot4e^{-0\cdot4}$ $x = 0\cdot737$ distance moved $1\cdot5 - 0\cdot737 = 0\cdot763$	**1**
Notes:					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
16.	(a)	(i)	•¹ sketch graph showing speed increase/decrease of both runners and annotation of meeting after 3 seconds •² sketch complete with relevant annotation	•¹ •² 	**2**
		(ii)	•³ use equations of motion under constant acceleration to find time for deceleration of P	$s=\ t=\ u=12\ \ v=9\ \ a=4$ •³ $v=u+at$ $9=12-4t$ $t=0\cdot75$ s	**1**
Notes: 1. Must show v/t graph beyond t=3 and a maximum speed for Q of 12ms^{-1} 2. Graph Q .. allow variations after t=3s but maximum speed must not exceed 12ms^{-1} as constant acceleration is not specified.					
Commonly Observed Responses:					
16.	(b)		•⁴ expression for area under the graph for *P* •⁵ correct displacement •⁶ find displacement for *Q* in three seconds •⁷ explain displacements •⁸ calculate distance	•⁴ $P:\ 27+\frac{1}{2}(2\cdot25+3)\times3$ •⁵ $34\cdot875$ metres •⁶ $\frac{1}{2}\times3\times9=13\cdot5$ metres •⁷  •⁸ $34\cdot875+0\cdot8-13\cdot5=22\cdot175$m	**5**
Notes:					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
17.			•¹ use $F = ma$ with substitution of $\frac{dv}{dt}$ for acceleration •² equate Impulse with change in Momentum •³ separate variables and start integration •⁴ use initial conditions with substitution •⁵ complete proof	•¹ $m\frac{dv}{dt} = -kv^2$ •² $I = mv$ •³ $\int \frac{mdv}{v^2} = -kt + c$ •⁴ $t = 0 \quad v = \frac{I}{m}$ $\frac{-m}{v} = -kt - \frac{m^2}{I}$ •⁵ $\frac{m}{v} = \frac{ktI + m^2}{I}$ $v = \frac{mI}{ktI + m^2}$	**5**

Notes:

1. Use of $c = \frac{m^2}{I}$ may appear in •⁴

Commonly Observed Responses:

[END OF MARKING INSTRUCTIONS]