2013 Applied Mathematics – Mechanics

Advanced Higher

Finalised Marking Instructions

Part One: General Marking Principles for Applied Mathematics – Statistics – Advanced Higher

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this Paper. These principles must be read in conjunction with the specific Marking Instructions for each question.

(a) Marks for each candidate response must <u>always</u> be assigned in line with these general marking principles and the specific Marking Instructions for the relevant question. If a specific candidate response does not seem to be covered by either the principles or detailed Marking Instructions, and you are uncertain how to assess it, you must seek guidance from your Team Leader/Principal Assessor.

(b) Marking should always be positive ie, marks should be awarded for what is correct and not deducted for errors or omissions.

GENERAL MARKING ADVICE: Applied Mathematics – Statistics – Advanced Higher

The marking schemes are written to assist in determining the "minimal acceptable answer" rather than listing every possible correct and incorrect answer. The following notes are offered to support Markers in making judgements on candidates' evidence, and apply to marking both end of unit assessments and course assessments.

These principles describe the approach taken when marking Advanced Higher Applied Mathematics papers. For more detailed guidance please refer to the detailed Marking Instructions.

1 The main principle is to give credit for the skills demonstrated and the criteria met. Failure to have a correct method may not preclude a candidate gaining credit for their solution.

2 The answer to one part of a question, even if incorrect, can be accepted as a basis for subsequent dependent parts of the question.

3 The following are not penalised:

- working subsequent to a correct answer (unless it provides firm evidence that the requirements of the question have not been met)
- legitimate variation in numerical values / algebraic expressions.

4 Full credit will only be given where the solution contains appropriate working. Where the correct answer might be obtained by inspection or mentally, credit may be given.

5 Sometimes the method to be used in a particular question is explicitly stated; no credit will be given where a candidate obtains the correct answer by an alternative method.

6 Where the method to be used in a particular question is not explicitly stated in the question paper, full credit is available for an alternative valid method. (Some likely alternatives are included but these should not be assumed to be the only acceptable ones.)

In the detailed Marking Instructions which follow, marks are shown alongside the line for which they are awarded. There are two codes used M and E. The code M indicates a method mark, so in question B3, **M1** means a method mark for understanding integration by parts. The code E refers to 'error', so in question B6(b), up to 2 marks can be awarded but 1 mark is lost for each error.

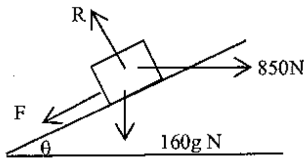
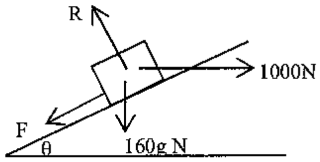
Part Two: Marking Instructions for each Question:

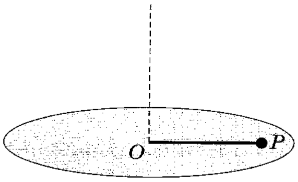
Section A

Question			Solution	Max Mark	Additional Guidance
A	**1**		**A particle is moving in a plane such that *t* seconds after the start of its motion, the velocity is given by $(3ti + 5t^2 j)$ ms^{-1}.** **The particle is initially at the point $(\frac{1}{2}i - 7j)$ metres relative to a fixed origin.** **Find the distance of the particle from *O* when $t = 3$** $s = \int \underline{v}dt = \frac{3}{2}t^2 i + \frac{5}{3}t^3 j + \underline{c}$ $t = 0 \quad \underline{s} = (\frac{1}{2}i - 7j) \quad \Rightarrow \quad \underline{c} = (\frac{1}{2}i - 7j)$ $\underline{s} = (\frac{3}{2}t^2 + \frac{1}{2})i + (\frac{5}{3}t^3 - 7)j$ t = 3 $\underline{s} = (14i + 38j)$ $\lvert s \rvert = \sqrt{14^2+38^2} = 40.5$ metres	**3**	**M1** Integration of velocity for displacement with correct integration **1** Evaluate *c* and give vector for displacement **1** Find vector when $t = 3$ and its magnitude
A	**2**		**A ball of mass 0.5kg is released from rest at a height of 10 metres above the ground.** **If the ball reaches 2.5 metres after its first bounce, calculate the size of the impulse exerted by the ground on the ball.** **Method 1:** $s = 10 \quad t = \quad u = 0 \quad v = ? \quad a = g$ $\downarrow v^2 = u^2 + 2as$ $u^2 = 20g$ $u = 14$ ms^{-1} $s = 2.5 \quad t = \quad u = \quad v = 0 \quad a = -g$ $\uparrow v^2 = u^2 + 2as$ $0 = v^2 - 5g$ $v = 7$ $\uparrow I = mv - mu$ $I = 0{\cdot}5\,(7 - (-14))$ $I = 10{\cdot}5$ Ns **Method 2:** Initial $E_P = 5g$ On impact: $\frac{1}{2}mu^2 = 5g \quad \Rightarrow \quad u^2 = 20g \quad \Rightarrow \quad u = 14\text{ms}^{-1}$ Final $E_P = \frac{5g}{4} \quad \Rightarrow \quad \frac{1}{2}mv^2 = \frac{5g}{4} \quad \Rightarrow v = \sqrt{5g} = 7\text{ms}^{-1}$ $\uparrow I = mv - mu$ $I = 0{\cdot}5\,(7 - (-14)$ $I = 10{\cdot}5$ Ns	**4**	**M1** Motion under gravity to find velocity on impact **1** Value of *u* **M1** Motion under gravity to find velocity of rebound **M1** Impulse momentum equation **M1** Energy equation for initial PE and impact KE **1** Value of *u* **M1** Energy equation for final PE and rebound KE **M1** Impulse momentum equation

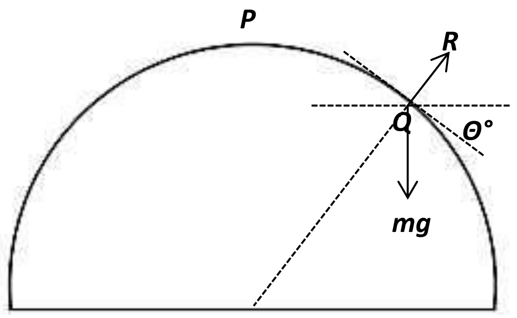
Question			Solution	Max Mark	Additional Guidance
A	**3**		**A particle of mass 3 kilograms moves under the action of its own weight and a constant force $\boldsymbol{F} = (3\boldsymbol{i} - 5.4\boldsymbol{j})$ where $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit vectors in the horizontal and vertical directions respectively.** **Initially the particle has velocity $(2\boldsymbol{i} - \boldsymbol{j})\,\mathrm{ms}^{-1}$ as it passes through a point *A*. The particle passes through *B* after 4 seconds. Find the work done to move the particle from *A* to *B*.**	**5**	
			Method 1: $F = ma$ $\begin{pmatrix}3\\5\cdot 4\end{pmatrix}+\begin{pmatrix}0\\-3g\end{pmatrix}=3a \Rightarrow a=\begin{pmatrix}1\\-8\end{pmatrix}$		**M1** Collective force correct **M1** Method and calculation of acceleration
			$s = ut + \frac{1}{2}at^2$ $s=\begin{pmatrix}2\\-1\end{pmatrix}\times 4+\frac{1}{2}\begin{pmatrix}1\\-8\end{pmatrix}\times 4^2=\begin{pmatrix}16\\-68\end{pmatrix}$		**1** Use of *stuva* and correct substitution to find displacement
			Work done $= F \bullet s = \begin{pmatrix}3\\-24\end{pmatrix} \bullet \begin{pmatrix}16\\-68\end{pmatrix} = 48 + 1632 = 1680J$		**M1** Method and calculation of work done **1** Correct answer
			Method 2 $F = ma$ $\begin{pmatrix}3\\-5.4\end{pmatrix}+\begin{pmatrix}0\\-3g\end{pmatrix}=3a \Rightarrow a=\begin{pmatrix}1\\-8\end{pmatrix}$		**M1** Collective force correct **M1** Method and calculation of acceleration
			$\underline{a} = i - 8j$ $\underline{v} = ti - 8tj + c$ $t = 0 \;\; \underline{v} = 2i - j \Rightarrow \underline{v} = (2 + t)i - (8t + 1)\,j$		**1** Integration to find expression for $\underline{\boldsymbol{v}}$
			Work done $= \int_0^4 \boldsymbol{F} \cdot \boldsymbol{v}\,\boldsymbol{dt} = \int_0^4 \begin{pmatrix}3\\-24\end{pmatrix} \cdot \begin{pmatrix}2+\boldsymbol{t}\\-8\boldsymbol{t}-1\end{pmatrix}\boldsymbol{dt}$ $= \int_0^4 (6 + 3\boldsymbol{t} + 192\boldsymbol{t} + 24)\boldsymbol{dt} = \int_0^4 (195\boldsymbol{t} + 30)\boldsymbol{dt}$ $= \left[\frac{195}{2}\boldsymbol{t}^2 + 30\boldsymbol{t}\right]_0^4 = 1680$		**M1** Method and calculation of work done **1** Correct answer

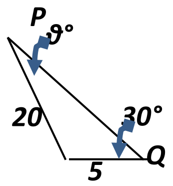
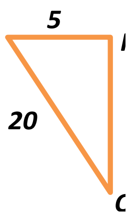
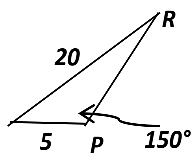
Question			Solution	Max Mark	Additional Guidance
A	4		**A go-kart of mass 100 kilograms accelerates at $3ms^{-2}$ at the instant when its speed is $5ms^{-1}$ and the engine's power is at a maximum.** **Given that there is a total resistance to motion of 60N throughout the go-kart's motion, find the maximum speed which the go-kart can achieve.** $F = \frac{P}{v} = \frac{P}{5}$ Accelerating force = $\frac{P}{5} - 60$ $F = ma \Rightarrow \frac{P}{5} - 60 = 100 \times 3$ $P = 1800W$ Maximum speed: $a = 0 \Rightarrow \frac{P}{V_{max}} - 60 = 0$ $V_{max} = \frac{P}{60} = \frac{1800}{60} = 30ms^{-1}$	4	**M1** Correct formula and substitute to find accelerating force **1** Calculation of Power **M1** Understanding of maximum speed **1** Calculation of speed.

Question			Solution	Max Mark	Additional Guidance
A	5		**A piano of mass 160 kilograms is resting on a rough plane inclined at an angle θ° to the horizontal, where $\tan\theta = \frac{7}{24}$.** **When a removal man applies a horizontal force of 850 newtons, the piano is just on the point of moving up the plane. Find the value of the coefficient of friction between the piano and the surface of the plane** **When the removal man increases the horizontal force to 1000 newtons, the piano begins to accelerate up the plane, along the line of greatest slope. How far does the piano travel in 3 seconds?**	6	
					**M1** Correct diagram including friction, horizontal force, normal reaction and weight and method of resolving in 2 perpendicular directions
			$\nwarrow$ $R = 850\sin\theta + 160g\cos\theta$ $= 850(\frac{7}{25}) + 160(9\cdot8)(\frac{24}{25}) = 1743\cdot28\text{N}$		**1** Correct resolution perpendicular to the slope
			$\nearrow$ $850\cos\theta = \mu R + 160g\sin\theta$ $\mu R = 850(\frac{24}{25}) - 160(9\cdot8)(\frac{7}{25}) = 376\cdot96\text{N}$		**1** Correct resolution parallel to slope
			$\mu = \frac{\mu R}{R} = \frac{376\cdot96}{1743\cdot28} = 0\cdot216$		**1** Correct value of μ
			 $\nwarrow$ $R = 1000(\frac{7}{25}) + 160(9\cdot8)(\frac{24}{25}) = 1785\cdot28\text{N}$ $\nearrow$ $F = ma$: $1000(\frac{24}{25}) - 160(9\cdot8)(\frac{7}{25}) - \mu(1785\cdot28) = 160a$ $a = 0\cdot846\text{ms}^{-2}$		**1** Equilibrium perpendicular to slope and $F = ma$ along the slope to find acceleration
			$s = ?\quad t = 3\quad u = 0\quad a = 0\cdot846$ $s = ut + \frac{1}{2}at^2$: $s = \frac{1}{2}(0\cdot846)(3^2) = 3\cdot81$ metres		**1** *stuva* substitution to find displacement

Question			Solution	Max Mark	Additional Guidance
A	6		**A rough disc rotates in a horizontal plane with a constant angular velocity ω about a fixed vertical axis through the centre *O*. A particle of mass *m* kilograms lies at a point *P* on the disc and is attached to the axis by a light elastic string *OP* of natural length *a* metres and modulus of elasticity 2*mg*.**  **The particle is at a distance of $\frac{5a}{4}$ from the axis and the coefficient of friction between *P* and the disc is $\frac{3}{20}$. Find the range of values for ω such that the particle remains stationary on the disc.**	5	
			$T = \frac{\lambda x}{l} = \frac{2mg(a/4)}{a} = \frac{mg}{2}$		**M1** Hooke's Law
			Slipping out: $\uparrow R = mg$ $\leftarrow\ T + \mu R = mr\omega^2$ $\frac{mg}{2} + \frac{3mg}{20} = m\left(\frac{5a}{4}\right)\omega^2$ $\frac{13mg}{20} = \frac{5ma\omega_1^2}{4}$		**M1** vertically equilibrium **<u>and</u>** horizontally combines forces of elastic string and friction
			$\omega_1 = \sqrt{\frac{13g}{25a}}$		**1** Correct value for ω_1
			Slipping in: $\leftarrow\ T - \mu R = mr\omega_2^2$ $\frac{mg}{2} - \frac{3mg}{20} = m\left(\frac{5a}{4}\right)\omega_2^2$		**M1** Correct interpretation for slipping in
			$\omega_2 = \sqrt{\frac{7g}{25a}}$ No slipping if: $\sqrt{\frac{7g}{25a}} \le \omega \le \sqrt{\frac{13g}{25a}}$		**1** Calculation of ω_2 and final statement

Question			Solution	Max Mark	Additional Guidance
A	**7**		**A light elastic string of natural length *l* metres hangs from a fixed point *O* with a particle of mass *m* kilograms attached at its lower end. In equilibrium the string is extended by *e* metres.** **The particle is then pulled down a further distance *a* metres where *a* < *e* and released.** **Show that the ensuing motion is simple harmonic and state the period of the motion.** **The maximum velocity of the particle during motion is** $\frac{1}{2}\sqrt{ge}$ **.** **Find an expression for the amplitude of the motion in terms of *e*.**	**6**	
			In equilibrium: $Tension = \frac{\lambda e}{l} = mg \quad \lambda = \frac{mgl}{e}$		**M1** Use of Hooke's Law in equilibrium position
			$mg - T = m\ddot{x}$ $mg - \frac{\lambda(e+x)}{l} = m\ddot{x}$		**M1** In extension $\downarrow$*F=ma* **<u>and</u>** substitution
			$\omega = -\sqrt{\frac{\lambda}{ml}} \quad mg - \frac{mgl}{e}(\frac{e+x}{l}) = m\ddot{x}$ $\ddot{x} = \frac{-g}{e}(e+x-e)$ $\ddot{x} = -\frac{g}{e}x \quad [\ddot{x} = -\frac{\lambda}{ml}x]$		**1** Proof of SHM
			i.e SHM where $\omega = \sqrt{\frac{g}{e}}$		**M1** Statement of Period
			Period $= \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{e}{g}} \quad [2\pi\sqrt{\frac{ml}{\lambda}}]$		**M1** Statement for v_{max} and substitution
			$v_{max} = aw$ $\frac{1}{2}\sqrt{ge} = a\sqrt{\frac{g}{e}}$ $\frac{1}{4}ge = a^2(\frac{g}{e})$ $a = \frac{1}{2}e$		**1** Calculation of final answer

Question			Solution	Max Mark	Additional Guidance
A	8		**A smooth solid hemisphere of radius *a* metres is fixed with its plane face on a horizontal table and its curved surface uppermost. A particle *P* of mass *m* kilograms is placed at the highest point on the hemisphere and given an initial horizontal speed $\sqrt{\frac{ag}{2}}$ ms^{-1}. The particle moves along the curved surface of the hemisphere until it leaves the surface at *Q*. Calculate the angle between the tangent at *Q* and the horizontal, and find an expression for the speed of the particle at *Q*.** 	6	
			Energy at Initial position: $E_P + E_K = mga + \frac{1}{2}m\frac{ga}{2} = \frac{5ga}{4}$		**M1** initial total energy stated
			At ***Q***: Total energy: $E_P + E_K = mga\cos\theta + \frac{1}{2}mv^2$ Conservation of energy: $mga\cos\theta + \frac{1}{2}mv^2 = \frac{5mga}{4}$ $v^2 = \frac{5ga}{2} - 2ga\cos\theta$ (i)		**M1** Energy at ***Q*** and conservation of energy
			At *Q* consider forces acting towards *O* $mg\cos\theta - R = \frac{mv^2}{a}$		**M1** Apply *F*=*ma* towards ***O***
			When body leaves surface $R = 0$ $mg\cos\theta = \frac{mv^2}{a}$		**M1** Interpretation of body leaving surface as *R* = 0 (stated)
			$v^2 = ga\cos\theta$ (ii) In (i) $\frac{5ga}{2} - 2ga\cos\theta = ga\cos\theta$		**1** Algebraic manipulation to find θ
			$\Rightarrow \cos\theta = \frac{5}{6} \Rightarrow \theta = 33{\cdot}6^{\circ}$ $v = \sqrt{\frac{5ga}{6}}$		**1** Substitution in (ii) to find *v*

Question			Solution	Max Mark	Additional Guidance
A	9		**A speedboat has to round three buoys *P*, *Q* and *R* as part of a race, starting at *P and travelling anticlockwise*.. The buoys are 200 metres from each other with *R* due North of *Q* and *P* lying to the west of the line QR. In still water, the speedboat travels at 20ms^{-1}. The water current is steady at 5ms^{-1} flowing from due West.** **Find the mean speed for one complete lap of the course.**	9	
			PQ: 		**M1** Interpretation of journey ***PQ*** /annotated diagram
			$V_C = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \quad \frac{5}{\sin\theta} = \frac{20}{\sin 30°}$ $\theta = 7\cdot2° \Rightarrow \alpha = 142\cdot8°$ $V_{PQ}{}^2 = 20^2 + 5^2 - 2\times20\times5\cos142\cdot8°$ $V_{PQ} = 24\cdot2$		**1** Calculation of true velocity ***PQ***
			$T_{PQ} = \frac{200}{V_{PQ}} = 8\cdot3$ secs		**1** Time for ***PQ***
			QR: 		**M1** Interpretation of ***QR***/annotated diagram
			$V_{QR} = \sqrt{20^2 - 5^2} = 19\cdot4$ $T_{QR} = \frac{200}{V_{QR}} = 10\cdot3$ secs		**1** Calculation of true velocity ***QR*** ***and*** time for ***QR***
			RP:  $\frac{5}{\sin\theta} = \frac{20}{\sin150°}$ $\theta = 7\cdot2° \Rightarrow \alpha° = 22\cdot8°$		**M1** Interpretation of ***RP*** /annotated diagram (this is more demanding hence repeated method mark)
			$V_{RP}{}^2 = 20^2 + 5^2 - 2\times20\times5\cos22\cdot8°$ $V_{RP} = 15\cdot5$ $T_{RP} = \frac{200}{15\cdot5} = 12\cdot9\sec s$		**1** Calculation of true velocity ***RP*** ***and*** time for ***RP***
			Total Time for lap: 8·3+10·3+12·9 = 31·5 secs		**1** Total Time
			Mean Speed per lap: $\frac{600}{31\cdot5} \approx 19ms^{-1}$		**1** Mean speed

Question			Solution	Max Mark	Addition Guidance
A	10		**Two projectiles are launched simultaneously from points A and B, where B is due East of A and situated on the same horizontal plane through A. The projectile launched from point A is projected towards B with speed 90ms^{-1} at an angle of 30° to the horizontal. The projectile from point B is projected towards A with speed 50ms^{-1} at an angle θ° to the horizontal.** **The two missiles collide in mid-air at a distance *d* metres horizontally from point A.** **Show that the height *h* at this point of collision is** $h=\dfrac{d(4050\sqrt{3}-gd)}{12150}$ **Find the angle of projection θ° at which the projectile at B is launched.** **The projectiles collide 5 seconds after launch. Calculate the distance between A and B.**	10	
			Projectile A: → $d=90\cos 30°=45\sqrt{3}\times t$ $t=\dfrac{d}{45\sqrt{3}}=\dfrac{\sqrt{3}d}{45}$		**M1** Horizontal motion with constant speed to give expression for *t*
			$\uparrow\ s=h \quad t=t \quad u=90\sin 30°=45 \quad a=-g$ $s=ut+\dfrac{1}{2}at^2:\quad h=45(\dfrac{\sqrt{3}d}{135})-\dfrac{g}{2}(\dfrac{\sqrt{3}d}{135})^2$		**M1** Vertical motion under gravity with values for *stuva* stated
			$h=\dfrac{\sqrt{3}d}{3}-\dfrac{gd^2}{12150}$		**1** Expression for *h*
			$h=\dfrac{d(4050\sqrt{3}-gd)}{12150}$		**1** Manipulation to give answer
			Projectile B ↑: $s=\dfrac{d(4050\sqrt{3}-gd)}{12150}\quad t=\dfrac{\sqrt{3}d}{145}\quad u=50\sin\theta\quad a=-g$ $s=ut+\dfrac{1}{2}at^2$		**M1** Vertical motion under gravity with values for *stuva* stated
			$\dfrac{d(4050\sqrt{3}-gd)}{12150}=50\sin\theta(\dfrac{\sqrt{3}d}{145})-\dfrac{g}{2}(\dfrac{\sqrt{3}d}{145})^2$ $\dfrac{(4050\sqrt{3}-gd)}{12150}=\dfrac{10\sqrt{3}d}{27}\sin\theta-\dfrac{gd}{12150}$		**2** Algebraic substitution and manipulation
			$\sin\theta=\dfrac{9}{10}=0\cdot 9\Rightarrow\theta=64\cdot 2°$		**1** Expression for $\sin\theta$ and value of θ
			Horizontal displacements: $x_A=d=45\sqrt{3}t=225\sqrt{3}\quad[\approx 389\cdot 7]$ $x_B=50\cos\theta\times t=50\times\dfrac{\sqrt{19}}{10}\times 5=25\sqrt{19}\quad[\approx 109\cdot 0]$		**M1** Expressions for horizontal distances
			Distance between A and B $=225\sqrt{3}+25\sqrt{19}\approx 500\,\text{m}$		**1** Final answer

Question			Solution	Max Mark	Addition Guidance
A	11		**A body of fixed mass m kilograms is projected vertically upwards from a point on the surface of a planet with an initial speed of u ms^{-1}. Assuming that the gravitational force on the body is $\frac{GMm}{d^2}$ where d metres is the distance from the centre of the planet, show that the speed of the body when it has reached a height h metres above the surface is given by $v=\sqrt{u^2-\frac{2GMh}{R(R+h)}}$, where M kilograms is the mass of the planet, R metres is the radius of the planet, and G is the gravitational constant. Find an expression for the maximum height H reached by the body.** **Show that the escape speed necessary for the body to continue into space can be written in the form $u=k\sqrt{\frac{GM}{R}}$ and state the value of k.**	10	
			$F=ma$: $\frac{-GMm}{(R+h)^2}=m\times acc\to\frac{-GM}{(R+h)^2}=v\frac{dv}{dh}$		**M1** Use of $F=ma$ and appropriate substitution
			$\int\frac{-GM}{(R+h)^2}dh=\int vdv$		**M1** Method of separate variables
			$\frac{GM}{R+h}=\frac{v^2}{2}+c$		**1** Integration and substitution
			$h=0$, $v=u$: $\frac{GM}{R}=\frac{u^2}{2}+c\ \to\ c=\frac{GM}{R}-\frac{u^2}{2}$		**1** Expression for c using initial conditions
			$\frac{GM}{R+h}=\frac{v^2}{2}+\frac{GM}{R}-\frac{u^2}{2}$		
			$v^2=u^2+\frac{2GM}{(R+h)}-\frac{2GM}{R}=u^2+\frac{2GM(R-(R+h))}{R(R+h)}$		
			$v^2=u^2-\frac{2GMh}{R(R+h)}$		**1** Rearrangement of formula
			Max height: $v=0; h=H$		
			$u^2-\frac{2GMH}{R(R+H)}=0\ \to\ u^2=\frac{2GMH}{R(R+H)}$		**M1** Interpretation of max ht by substituting $v=0$
			$u^2R(R+H)=2GMH$		
			$u^2R^2+u^2RH=2GMH\to u^2R^2=H(2GM-u^2R)$		**1** Algebraic manipulation
			$H=\frac{R^2u^2}{2GM-Ru^2}$		**1** Correct answer
			Escape speed: $H\to\infty\Rightarrow 2GM-Ru^2=0$		**M1** Understanding of escape speed with substitution
			$u=\sqrt{\frac{2GM}{R}}\Rightarrow k=\sqrt{2}$		**1** Manipulation and value of k

Section B

Question			Sample Answer/Work	Max Mark	Criteria for Mark
B	**1**		**Given that $y = \sin(e^{5x})$, find $\frac{dy}{dx}$** $\frac{dy}{dx} = \cos e^{5x} \times \frac{d}{dx}(e^{5x})$ $= \cos e^{5x} \times 5e^{5x}$ $= 5e^{5x} \cos e^{5x}$	**2**	**1** First application of chain rule. **1** Second application of chain rule.
Notes:					

Question			Sample Answer/Work	Max Mark	Criteria for Mark
B	**2**		**Matrices are given as** **A** = $\begin{pmatrix} 4 & x \\ 0 & 2 \end{pmatrix}$ **B** = $\begin{pmatrix} 5 & 1 \\ 0 & 1 \end{pmatrix}$ **C** = $\begin{pmatrix} y & 3 \\ -1 & 2 \end{pmatrix}$		
B	**2**	**a**	**Write A^2 – 3B as a single matrix**	**2**	
			$A^2 - 3B = \begin{pmatrix} 4 & x \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 4 & x \\ 0 & 2 \end{pmatrix} - 3\begin{pmatrix} 5 & 1 \\ 0 & 1 \end{pmatrix}$		
			$= \begin{pmatrix} 16 & 6x \\ 0 & 4 \end{pmatrix} - 3\begin{pmatrix} 5 & 1 \\ 0 & 1 \end{pmatrix}$		**1** A^2 correct.
			$= \begin{pmatrix} 16 & 6x \\ 0 & 4 \end{pmatrix} - \begin{pmatrix} 15 & 3 \\ 0 & 3 \end{pmatrix}$		
			$= \begin{pmatrix} 1 & 6x-3 \\ 0 & 1 \end{pmatrix}$		**1** For correct evaluation of 3B and simplify.
B	**2**	**b**	**(i) Given that C is non-singular, find C^{-1}, the inverse of C.**	**2**	
			det C = 2y + 3		**1** Determinant correct.
			$C^{-1} = \dfrac{1}{2y+3}\begin{pmatrix} 2 & -3 \\ 1 & y \end{pmatrix}$		**1** Inverse correct.
B	**2**	**b**	**(ii) For what value of *y* would matrix C be singular?**	**1**	
			2y + 3 = 0 for C singular		
			$y = -\dfrac{3}{2}$		**1** y value correct.

Notes:

Question			Sample Answer/Work	Max Mark	Criteria for Mark
B	**3**		**Use integration by parts to obtain** $\int \frac{\ln x}{x^3}\,dx$ **where $x > 0$**	**4**	
			$u = \ln x,\ dv = \frac{1}{x^3}dx$		
			$du = \frac{1}{x}dx,\ v = \int \frac{1}{x^3}dx$		**M1** Understand integration by parts.
			$v = -\frac{1}{2x^2}$		
			$I = \ln x. -\frac{1}{2x^2} - \int -\frac{1}{2x^2}.\frac{1}{x}\,dx$		**1** Integrates *dv* and substitutes correctly.
			$= -\frac{\ln x}{2x^2} + \frac{1}{2}\int \frac{dx}{x^3}$		**1** Correctly combines *v* and *du*.
			$= -\frac{\ln x}{2x^2} - \frac{1}{4x^2} + c$		**1** Correctly integrates second term.

Notes:

3.1 Treat omission of "+*c*" as bad form: do not penalise.

3.2 Negative indices for *x* equally acceptable.

Question			Sample Answer/Work	Max Mark	Criteria for Mark
B	4	a	**State the results for** $\sum_{r=1}^{n} r$ **and** $\sum_{r=1}^{n} r^3$ **in terms of** ***n*****.** **Hence show that** $\sum_{r=1}^{n}(r^3 - 3r) = \frac{n(n+1)(n-2)(n+3)}{4}$	4	
			$\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$ $\qquad \sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$		**1** Both formulae correct.
			$\sum_{r=1}^{n}(r^3 - 3r) = \sum_{r=1}^{n} r^3 - 3\sum_{r=1}^{n} r$		**1** Correct separation.
			$= \frac{n^2(n+1)^2}{4} - \frac{3n(n+1)}{2}$		**1** Substitution .
			$= \frac{n(n+1)}{4}[n(n+1) - 6]$		
			$= \frac{n}{4}(n+1)(n^2 + n - 6)$ **Note:** This or equivalent intermediate algebra required for this mark.		**1** Algebra correct.

Notes:

Question			Sample Answer/Work	Max Mark	Criteria for Mark
B	**4**		**(cont)**		
B	**4**	**b**	**Use the above result to evaluate** $\sum_{r=5}^{15}(r^3 - 3r)$	**2**	
			$\sum_{r=5}^{15}(r^3 - 3r) = \sum_{r=1}^{15}(r^3 - 3r) - \sum_{r=1}^{4}(r^3 - 3r)$		**1** Correct limits.
			$= \frac{15\times16\times18\times13}{4} - \frac{4\times5\times2\times7}{4}$		
			= 14 040 – 70		
			= 13 970		**1** Correct evaluation.

Notes:

Question			Sample Answer/Work	Max Mark	Criteria for Mark
B	5		**Find the general solution of the differential equation**	6	
			$\boldsymbol{\frac{1}{x}\frac{dy}{dx}+2y=6, x\neq 0}$		
			$\frac{dy}{dx}+2xy=6x$		**1** Multiplies through by *x*.
			I.F = $e^{\int 2x} = e^{x^2}$		**1** Correct integrating factor.
			$e^{x^2}\frac{dy}{dx} + e^{x^2}\,2xy = 6x \,.\, e^{x^2}$		
			$\frac{d}{dx}(e^{x^2}.y) = 6x \,.\, e^{x^2}$		**1** Recognises LHS as exact differential of g × I.F.
			$\int \frac{d}{dx}(e^{x^2}.y)\,dx = \int 6x \,.\, e^{x^2}\,dx$		**1** Knows to integrate.
			$e^{x^2} \,.\, y = 3e^{x^2} + c$		**1** Integrates correctly.[2]
			$y = 3 + \frac{c}{e^{x^2}}$		**1** Divides through by e^{x^2}.

Notes:
5.1 Final answer of $y = 3 + ce^{-x^2}$ also correct.
5.2 "+c" required for mark here.

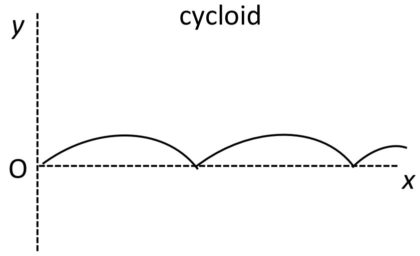
Question			Sample Answer/Work	Max Mark	Criteria for Mark
B	**5**		**Alternative:**		
			$\frac{1}{x}\frac{dy}{dx} = 6 - 2y$		
			$\frac{dy}{6-2y} = x\,dx$		**1** Separates variables.
			$-\frac{1}{2}\ln\lvert 6-2y\rvert = \frac{1}{2}x^2 + k$		**1** Integrates LHS. **1** Integrates RHS (constant on either side).
			$\ln\lvert 6-2y\rvert = -x^2 - 2k$		**1** Prepares for exponential.
			$6 - 2y = Ae^{-x^2}$		**1** Converts form to exponential.[3,4]
			$-2y = Ae^{-x^2} - 6$		
			$y = \frac{1}{2}Ae^{-x^2} + 3$		**1** Rearranges to make y subject.[3,5]
			$y = 3 + \frac{C}{e^{x^2}}$		

Notes:

5.3 Any constant acceptable. Therefore, term containing constant can be positive or negative.

5.4 $6 - 2y = e^{-x^2-c}$ a valid alternative for this mark.

5.5 Either of last two lines valid for award of final mark.

Question			Sample Answer/Work	Max Mark	Criteria for Mark
B	**6**		**The cycloid curve below is defined by the parametric equations** $\boldsymbol{x = t - \sin t,\ y = 1\text{-} \cos t.}$  		
B	**6**	**a**	**Find** $\frac{\mathbf{dy}}{\mathbf{dx}}$ **in terms of** ***t*** $\frac{dy}{dt} = \sin t,\ \ \frac{dx}{dt} = 1 - \text{cost}\ t$ $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{\sin\ t}{1 - \cos t}$	**2**	**1** Appropriate differentiation. **1** Correct use.
Notes:					

Question			Sample Answer/Work	Max Mark	Criteria for Mark
B	**6**		**(cont)**		
B	**6**	**b**	**Show that the value of $\frac{d^2y}{dx^2}$ is always negative, in the case where $0 < t < 2\pi$**	**5**	
			$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \div \frac{dx}{dt}$		**M1** Correct application of method.
			$= \frac{(1-\cos t)\cos t - \sin t(\sin t)}{(1-\cos t)^2} \div (1-\cos t)$		
			$= \frac{\cos t - \cos^2 t - \sin^2 t}{(1-\cos t)^3}$		**2E1** Differentiates /substitutes correctly.
			$= \frac{-[(\cos^2 t + \sin^2 t) - \cos t]}{(1-\cos t)^3}$		**1** Uses $\sin^2 t + \cos^2 t = 1$ and simplifies.
			$= \frac{-(1-\cos t)}{((1-\cos t))^3}$		
			$= -\frac{1}{(1-\cos t)^2} < 0$		**1** Clear explanation.
			Hence		
			$\frac{d^2y}{dx^2} < 0$, for $0 < t < 2\pi$		
Notes:					

Question			Sample Answer/Work	Max Mark	Criteria for Mark
B	**6**	**c**	**A particle follows the path of the cycloid where *t* is the time elapsed since the particle's motion commenced.** **Calculate the speed of the particle when** $t = \frac{\pi}{3}$. Speed = $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = 1$	**2**	**1** Correct formula. **1** Applies correct values to obtain a speed of 1.

Notes:

[END OF MARKING INSTRUCTIONS]