AH National Qualifications 2025

X802/77/11

Mathematics of Mechanics

WEDNESDAY, 7 MAY

1:00 PM – 4:00 PM

Total marks — 100

Attempt ALL questions.

You may use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate. Any rounded answer should be accurate to an appropriate number of significant figures unless otherwise stated.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you used this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not you may lose all the marks for this paper.

FORMULAE LIST

Newton's inverse square law of gravitation

$$F = \frac{GMm}{r^2}$$

Simple harmonic motion

$$v^2 = \omega^2(a^2 - x^2)$$

$$x = a\sin(\omega t + \alpha)$$

Centre of mass

Triangle: $\frac{2}{3}$ along median from vertex.

Semicircle: $\frac{4r}{3\pi}$ along the axis of symmetry from the diameter.

Coordinates of the centre of mass of a uniform lamina, area A square units, bounded by the equation $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$ is given by

$$\bar{x} = \frac{1}{A}\int_a^b xy\,dx \qquad \bar{y} = \frac{1}{A}\int_a^b \frac{1}{2}y^2\,dx$$

Standard derivatives	
$f(x)$	$f'(x)$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\ln x$	$\frac{1}{x}$
e^x	e^x

Standard integrals	
$f(x)$	$\int f(x)\,dx$
$\sec^2(ax)$	$\frac{1}{a}\tan(ax) + c$
$\frac{1}{x}$	$\ln\lvert x\rvert + c$
e^{ax}	$\frac{1}{a}e^{ax} + c$

Total marks — 100 marks

Attempt ALL questions

Note that g m s^{-2} denotes the magnitude of the acceleration due to gravity. Where appropriate, take its magnitude to be 9.8 m s^{-2}.

1. A projectile is launched from a point on horizontal ground. It has an initial speed of 20 m s^{-1} at an angle of 33° to the horizontal.

Find the range of the projectile. **3**

2. Differentiate $f(x) = e^{4x} \sec 3x$ **2**

3. A car of mass 1500 kg is travelling along a horizontal road. The car's engine is working at a constant rate of 18 000 W. The car experiences a constant resistance to motion of magnitude 450 N.

Calculate the acceleration of the car when it is travelling at 5 m s^{-1}. **3**

4. Use the substitution $u = \sin 5x$ to find $\int \sin^3 5x \cos 5x \, dx$. **3**

5. A yacht leaves a harbour and sails with a constant velocity of $\begin{pmatrix} 2.5 \\ 0 \end{pmatrix}$ m s^{-1}.

At the same instant, a motorboat is at a position $\begin{pmatrix} 1000 \\ 500 \end{pmatrix}$ metres, relative to the harbour, and is moving with a constant velocity of $\begin{pmatrix} 0 \\ -7 \end{pmatrix}$ m s^{-1}.

Determine the shortest distance between the yacht and the motorboat. **5**

6. A particle has position $x = 4t + 3$ metres and $y = t^2 + 5$ metres relative to an origin after t seconds.

Find the position of the particle at the instant when it is moving parallel to the line $y = x$. **3**

MARKS

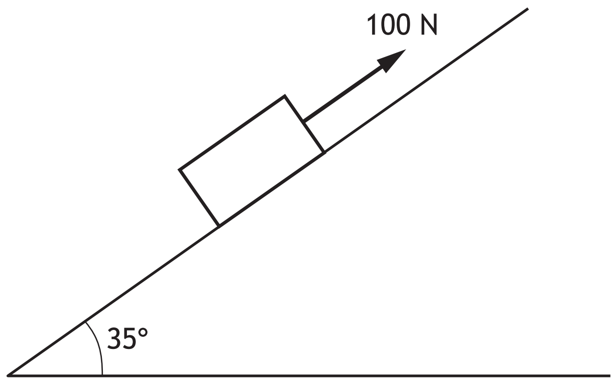
7. A motorbike is being tested. A rider accelerates the bike from rest in a horizontal straight line with an acceleration of $3t$ m s^{-2}, where t is the time in seconds from the start of the motion.

(a) Calculate the distance travelled by the bike at the instant the speed is measured as 24 m s^{-1}. **4**

The speed of the bike and the distance it travels are measured for a period of time, to allow performance data for the bike to be gathered.

(b) Explain why this acceleration can only be sustained for a short period of time. **1**

8. A box of mass 12 kg is held on a rough slope inclined at an angle of 35° to the horizontal, by a force of magnitude 100 newtons, acting parallel to and up the slope. The box is on the point of moving up the slope.



(a) Calculate the value of the coefficient of friction between the box and the slope. **4**

The force applied to the box is reduced so that the box is now on the point of slipping down the slope.

(b) Calculate the magnitude of this force. **2**

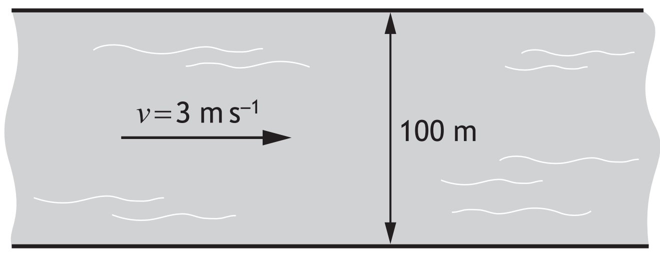
9. A curve is defined by the equation $y = x^2 + 2$ between $x = -3$ and $x = 3$.

The curve is rotated 2π radians around the x-axis.

Find the volume of revolution. **3**

MARKS

10. A river is 100 metres wide and the current flows at a rate of 3 m s^{-1}. A person in a boat which can travel at 5 m s^{-1} in still water, crosses the river to a point directly opposite their starting point on the bank.



(a) (i) Calculate the time it takes for the boat to cross the river. **2**

(ii) Find at what angle to the bank the person needs to steer the boat in order to complete the crossing. **1**

The person decides to make the return crossing, but this time wants to do so in the shortest time possible.

(b) Calculate the distance downstream that the boat is carried as a result. **2**

11. A spring of natural length 20 cm is suspended vertically from a fixed point.

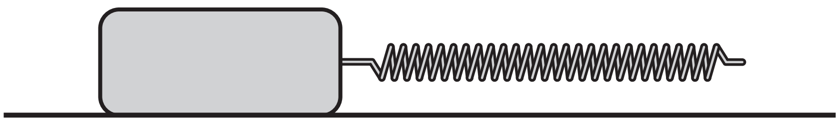
A mass of 1.5 kg is attached to the other end of the spring.

When the system is in equilibrium, the extension of the spring is 6 cm.

(a) Calculate the modulus of elasticity of the spring. **2**

The spring is detached from the fixed point, and the mass, still attached to the spring, is then placed on a rough, horizontal surface.

The spring is pulled horizontally.



When the extension of the spring reaches 4.8 cm, the mass is on the point of moving.

(b) Determine the coefficient of friction between the mass and the surface. **3**

[Turn over

MARKS

12. A block with mass 5 kg, at rest on a smooth horizontal surface, is given an impulse of 20 Ns.

The block moves onto a rough horizontal surface and eventually comes to rest again.

The coefficient of friction between the block and the rough surface is 0.3.

Calculate the distance the block travels on the **rough surface**. **3**

13. Solve the differential equation

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}-20y=0$$

given that $y=1$ and $\frac{dy}{dx}=22$ when $x=0$. **5**

14. A driverless car is tested on a straight track for a total time of $4T$ seconds.

It initially accelerates at a constant rate from rest to a speed of 30 m s^{-1} in a quarter of the total time.

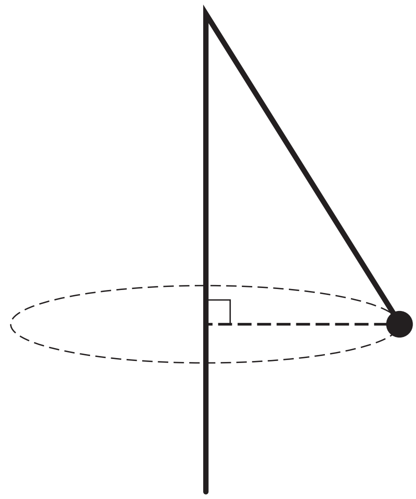
The car then maintains this speed for 40 seconds, before decelerating at a constant rate to a speed of 10 m s^{-1}, at which point the test ends.

(a) Sketch a velocity-time graph to illustrate the car's motion during this test. **2**

(b) Given that the total distance travelled is 2.5 km, calculate the total time. **3**

15. Use integration by parts to find $\int 9x^2e^{3x}\,dx$. **4**

16. A ball of mass 80 grams is attached to a light inextensible rope which is connected to a vertical pole. The rope is free to rotate around the pole.

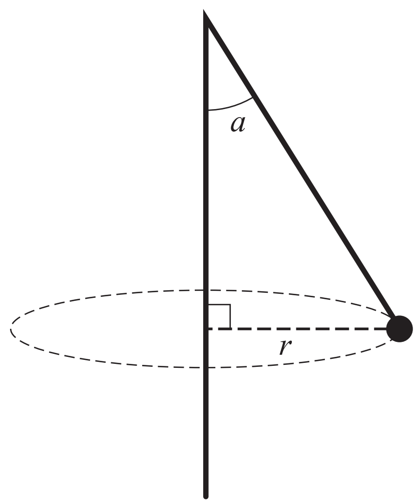


The ball has a speed of 7 m s^{-1} which causes it to move in a horizontal circle of radius 1.25 m around the pole.

(a) (i) Calculate the angle the rope makes with the pole. **3**

(ii) Calculate the tension in the rope. **1**

The length of the rope is shortened to 0.6 metres. The ball continues with the same speed of 7 m s^{-1} and describes a horizontal circle of radius r metres. The rope makes an angle of a with the pole.



(b) (i) Show that $\tan a = \dfrac{r}{\sqrt{0.36 - r^2}}$. **1**

(ii) Calculate the value of r. **3**

[Turn over

MARKS

17. (a) Find partial fractions for $\dfrac{12}{9-4x^2}$. **3**

A toy vehicle of mass 1.2 kg is pushed from rest across a horizontal floor by a force of magnitude 0.9 newtons. It experiences a variable resistive force of $0.4v^2$ newtons, where v m s^{-1} is the velocity of the toy after t seconds.

(b) (i) Show that $\dfrac{dv}{dt}=\dfrac{9-4v^2}{12}$. **1**

(ii) Using your answer to part (a), calculate the time it takes for the toy to reach a velocity of 0.9 m s^{-1}. **5**

18. A 5 kg object is attached between two identical horizontal springs of natural length 40 cm. The other ends of the springs are attached to fixings in a spacecraft with zero gravity. The fixings are 80 cm apart. The object is pulled a short distance towards one of the fixings and then released.

(a) Show that the object moves with simple harmonic motion modelled by the equation

$$\ddot{x}=-\lambda x,$$

where x metres is the displacement from the equilibrium position and λ newtons is the modulus of elasticity of the springs. **2**

The modulus of elasticity of the springs is 64 newtons.

(b) Find the period at which the object oscillates. **2**

The object is now pulled 6 cm towards one of the fixings and then released.

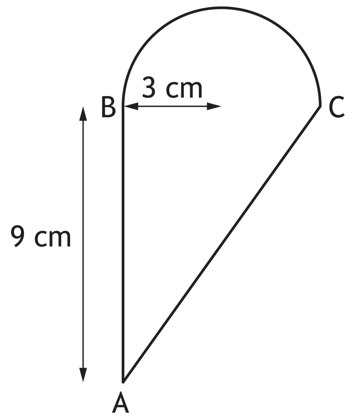
(c) Find the maximum velocity of the subsequent motion. **1**

A new object is placed between the springs and displaced slightly. The subsequent motion has a period of 1.2 seconds.

(d) Calculate the mass of this object. **2**

MARKS

19. A uniform lamina is made from a semicircle of radius 3 cm and a right angled triangle ABC of height 9 cm.



The point A is one vertex of the triangle.

(a) Taking A as the origin, find the coordinates of the centre of mass of the lamina, given that the line AB lies along the positive y-axis. **6**

The lamina is now freely suspended from point A.

(b) Calculate the angle that the line AB makes with the downward vertical. **1**

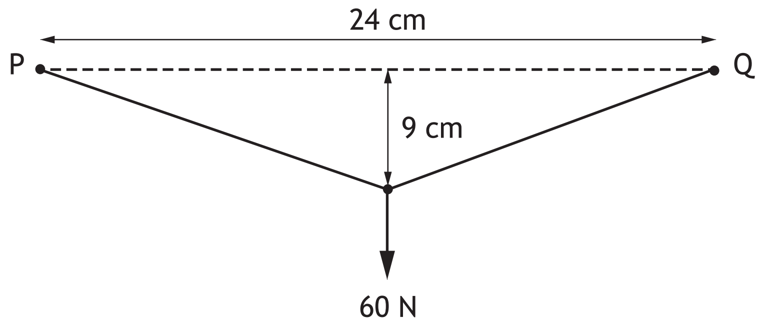
[Turn over

MARKS

20. A catapult is made from an elastic string of natural length 26 cm and modulus of elasticity λ newtons.

The two ends of the string are attached to fixed points P and Q, where PQ has length 24 cm. P and Q lie in the same horizontal plane.

A particle of weight 60 N rests in equilibrium in the middle of the string, at a distance of 9 cm below the line PQ.



(a) Calculate the value of λ. **4**

The particle is now pulled vertically downwards to a distance of 35 cm below PQ, and then released.

(b) (i) Calculate the elastic potential energy in the string before the particle is released. **1**

(ii) Assuming that all of this elastic potential energy is converted into kinetic and gravitational potential energy, calculate the speed of the particle at the point the string becomes slack. **4**

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