National Qualifications 2024

X802/77/11

Mathematics of Mechanics

FRIDAY, 17 MAY

12:30 PM – 3:30 PM

Total marks — 100

Attempt ALL questions.

You may use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate. Any rounded answer should be accurate to an appropriate number of significant figures unless otherwise stated.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not you may lose all the marks for this paper.

X8027711

A/HTP

FORMULAE LIST

Newton's inverse square law of gravitation

$$F = \frac{GMm}{r^2}$$

Simple harmonic motion

$$v^2 = \omega^2(a^2 - x^2)$$

$$x = a\sin(\omega t + \alpha)$$

Centre of mass

Triangle: $\frac{2}{3}$ along median from vertex.

Semicircle: $\frac{4r}{3\pi}$ along the axis of symmetry from the diameter.

Coordinates of the centre of mass of a uniform lamina, area A square units, bounded by the equation $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$ is given by

$$\bar{x} = \frac{1}{A}\int_a^b xy\,dx \qquad \bar{y} = \frac{1}{A}\int_a^b \frac{1}{2}y^2\,dx$$

Standard derivatives	
$f(x)$	$f'(x)$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\ln x$	$\frac{1}{x}$
e^x	e^x

Standard integrals	
$f(x)$	$\int f(x)\,dx$
$\sec^2(ax)$	$\frac{1}{a}\tan(ax) + c$
$\frac{1}{x}$	$\ln\lvert x\rvert + c$
e^{ax}	$\frac{1}{a}e^{ax} + c$

MARKS

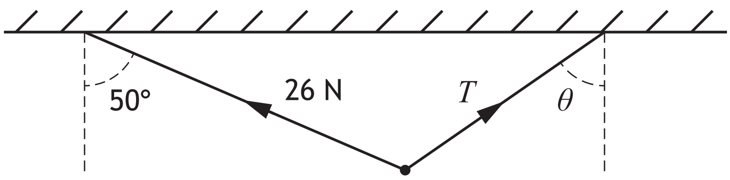
Total marks — 100 marks

Attempt ALL questions

Note that g m s^{-2} denotes the magnitude of the acceleration due to gravity. Where appropriate, take its magnitude to be 9.8 m s^{-2}.

1. A body of mass 8 kg is held in equilibrium by two light inextensible strings attached to a horizontal ceiling.

   The tensions in the strings are 26 newtons and T newtons, and the angles the strings make with the downward vertical are 50° and θ respectively, as shown in the diagram.



   Calculate the values of T and θ. **4**

2. (a) Find partial fractions for

$$\frac{7-2x}{(2x-1)(x+1)}$$ **3**

   (b) Hence find $\int \frac{7-2x}{(2x-1)(x+1)}\,dx$ **1**

3. Particle A has a mass of 30 grams and is travelling in a straight line with velocity u m s^{-1}.

   It collides with a stationary particle B and rebounds with a speed of $\frac{u}{3}$ m s^{-1} in the opposite direction.

   Particle B begins to move with a velocity of $\frac{u}{2}$ m s^{-1} in the original direction of motion.

   Calculate the mass of particle B. **3**

[Turn over

MARKS

4. Given $y = \dfrac{3x}{1+x^2}$, find $\dfrac{dy}{dx}$ and simplify your answer. **3**

5. A particle is moving with simple harmonic motion.

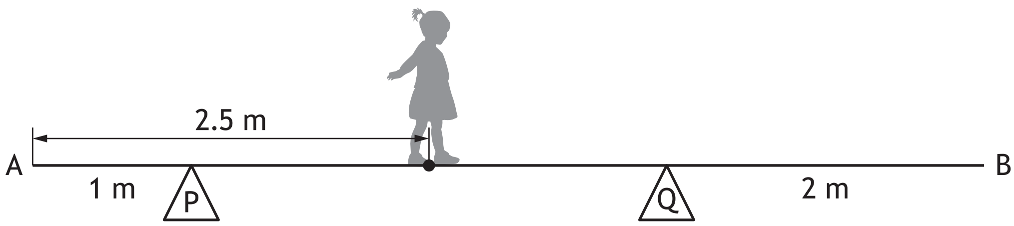
The maximum acceleration of the particle is 20 $m\,s^{-2}$ and its maximum speed is 10 $m\,s^{-1}$.

Calculate the speed of the particle when it is 1 metre from the centre of the oscillation. **4**

6. A function is defined on a suitable domain by $f(x) = \operatorname{cosec}^2(3x)$.

Evaluate $f'\left(\dfrac{\pi}{4}\right)$. **3**

7. A uniform beam AB, of length 6 metres and mass 45 kg, is placed on two supports at points P and Q, where AP is 1 metre and QB is 2 metres.



When a child of mass 22 kg stands on the beam at a distance of 2.5 metres from A, the beam rests horizontally in equilibrium, as shown in the diagram.

Calculate the magnitude of the reaction force at Q. **3**

MARKS

8. A particle is launched from an origin on horizontal ground and moves freely under gravity.

The particle is projected with speed u m s^{-1} at an angle θ to the horizontal.

(a) Show that the subsequent motion of the projectile has the equation

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

where x and y are measured in metres. **3**

A particle is fired at 20 m s^{-1} and it needs to pass over a wall which is 9 metres high; the base of the wall is 30 metres horizontally from the launch point.

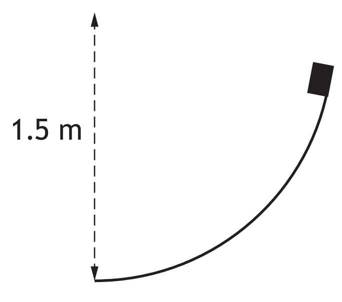
(b) Calculate the range of angles of projection which will allow the particle to pass over the wall.

Note that $\frac{1}{\cos^2\theta} = 1 + \tan^2\theta$. **4**

9. The velocity, v m s^{-1}, of a particle is defined by $3v + t^2e^v = 9$, where t is the time in seconds, and $t > 0$.

Use implicit differentiation to determine the instantaneous acceleration of the particle when $v = 0$. **5**

10. An object slides from rest down a smooth ramp which forms a circular arc of radius 1.5 metres.



The object travels a distance of 2.1 metres to reach the bottom of the ramp.

Determine the speed of the object after travelling this distance. **4**

[Turn over

MARKS

11. Solve the differential equation

$$\frac{dy}{dx} - 2y = 3e^{2x}$$

given that when $x = 0$, $y = 5$.

Express y in terms of x. **4**

12. A particle accelerates at $\frac{8}{3}$ m s^{-2} from rest for a time of 15 seconds. It then decelerates. The velocity of the particle during the deceleration is given by $v = 905e^{-0.20793t}$ m s^{-1}, where t is the time in seconds from the start of the motion.

The particle hits a barrier and stops after a total time of 35 seconds.

(a) Sketch a velocity-time graph for the particle. Show clearly all important information. **2**

(b) Determine the total distance the particle travels during its motion. **4**

13. The acceleration of a particle is given by the differential equation $\frac{dv}{dt} = \frac{2v}{1+t}$, where v m s^{-1} is its velocity and t is the time in seconds.

Given that the initial velocity of the particle is 2 m s^{-1}, calculate the velocity of the particle after 3 seconds. **4**

14. (a) State the integral of $3\sec 3x \tan 3x$ with respect to x. **1**

(b) Hence use integration by parts to find $\int 3\sin^2 3x \sec 3x \tan 3x \, dx$. **2**

MARKS

15. A motorbike and a car are at rest and beside each other. The car moves off first with uniform acceleration of 0.8 m s^{-2}.

(a) Find the distance the car has travelled after 5 seconds. **1**

The motorbike then accelerates uniformly in the same direction at 1.8 m s^{-2}.

(b) Find the time it takes for the motorbike to accelerate in order to draw level with the car. **4**

At the point when the motorbike and car are level, they see a sign for a road junction 200 metres ahead.

(c) If the driver of the car takes 0.8 seconds to react, find the necessary deceleration if the car is to stop at the junction. **3**

16. A curve is defined parametrically by $x = e^{3t} - e^{2t}$, $y = e^{3t} + e^{2t}$.

Find the coordinates of the point where the gradient of the curve is 2. **4**

17. A satellite is in a circular orbit about a planet of radius R metres.

The period of the orbit of the satellite is 2000π seconds.

The distance of the satellite from the centre of the planet is pR metres, where p is a constant.

Show that

$$R = \frac{1000^2 a}{p^3}$$

where a is the acceleration in m s^{-2} due to gravity at the surface of the planet. **5**

[Turn over

MARKS

18. A box is placed on a rough plane inclined at 10° to the horizontal.

The total mass of the box and its contents when full is 60 kg.

The box can be moved up the plane with constant velocity by a force of magnitude P newtons parallel to the slope.

When the box is empty, it has a mass of 40 kg and can be moved down the plane with constant velocity by a force of magnitude Q newtons.

Q acts in the opposite direction to P and $P = 5Q$.

(a) Show that the value of the coefficient of friction between the box and the plane is 0.327 to three significant figures. **5**

The full box is now attached to a light inextensible cable.

Starting from rest, the box is pulled up the plane by the cable. The tension in the cable is of magnitude 300 newtons and acts parallel to the slope.

After 10 seconds the cable snaps and the box continues to move until it comes to rest.

(b) Calculate the total distance the box travels up the plane. **6**

19. A boat Q is moving with a speed of $18\ \text{km h}^{-1}$ on a bearing of 270°.

To the captain of Q, another boat P appears to be moving at $20\ \text{km h}^{-1}$ on a bearing of 015°.

(a) Determine:

(i) the actual speed of P **3**

(ii) the direction in which P is moving. **1**

At 12 noon, Q is 12 km from P on a bearing of 290°.

Both boats continue on their current paths and at the same speeds.

(b) Find the time at which they will pass closest to each other in their subsequent journeys. **5**

MARKS

20. A motorised sledge of mass m kg is travelling along a rough horizontal track with instantaneous acceleration a m s^{-2}. The engine is working at a constant rate of P watts and the coefficient of friction is 0.1.

(a) When the instantaneous velocity of the sledge is V m s^{-1}, show that

$$P = mV\left(a + 0.1g\right)$$ **2**

The sledge now ascends a slope of 30° to the horizontal with the same coefficient of friction.

The engine now works at a rate of $3P$ watts. At a particular instant, both the instantaneous acceleration a m s^{-2} and the instantaneous velocity V m s^{-1} have the same values as they did when the sledge was travelling on the horizontal track.

(b) Calculate the acceleration at this instant. **4**

[END OF QUESTION PAPER]

[BLANK PAGE]

DO NOT WRITE ON THIS PAGE

[BLANK PAGE]

DO NOT WRITE ON THIS PAGE

[BLANK PAGE]

DO NOT WRITE ON THIS PAGE