



National
Qualifications
2019

X702/77/11

**Mathematics
of Mechanics**

THURSDAY, 25 APRIL

1:00 PM – 4:00 PM

Total marks — 100

Attempt ALL questions.

You may use a calculator.

Full credit will be given only to solutions that contain appropriate working.

State the units for your answer where appropriate. Any rounded answer should be accurate to three significant figures (or one decimal place for angles in degrees) unless otherwise stated.

Write your answers clearly in the answer booklet provided. In the answer booklet you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.



* X 7 0 2 7 7 1 1 *

FORMULAE LIST

Newton's inverse square law of gravitation

$$F = \frac{GMm}{r^2}$$

Simple harmonic motion

$$v^2 = \omega^2(a^2 - x^2)$$

$$x = a \sin(\omega t + \alpha)$$

Centre of mass

Triangle: $\frac{2}{3}$ along median from vertex.

Semicircle: $\frac{4r}{3\pi}$ along the axis of symmetry from the diameter.

Standard derivatives	
$f(x)$	$f'(x)$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\ln x$	$\frac{1}{x}$
e^x	e^x

Standard integrals	
$f(x)$	$\int f(x) dx$
$\sec^2(ax)$	$\frac{1}{a} \tan(ax) + c$
$\frac{1}{x}$	$\ln x + c$
e^{ax}	$\frac{1}{a} e^{ax} + c$

Total marks — 100

Attempt ALL questions

Candidates should observe that $g \text{ m s}^{-2}$ denotes the magnitude of the acceleration due to gravity. Where appropriate, take its magnitude to be 9.8 m s^{-2} .

1. A body of mass 4 kg is moving with initial velocity $(3\mathbf{i} + 2\mathbf{j}) \text{ m s}^{-1}$. It is given an impulse of $(6\mathbf{i} + \mathbf{j}) \text{ N s}$.
Calculate the magnitude of the final velocity and the angle it makes with the x -axis. 4

2. (a) If $f(x) = xe^{-3x}$, find the exact value of $f'(-1)$. 3

- (b) Given $g(t) = \frac{3t}{(2t+1)^2}$, find $g'(t)$, simplifying your answer. 3

3. A radio-controlled model boat moves from an origin with velocity $\mathbf{v}(t) = (4\mathbf{i} + (t+1)\mathbf{j}) \text{ m s}^{-1}$, where t is measured in seconds. The radio signal has a range of 80 metres.
Determine whether the boat is still within range of the radio signal after 10 seconds. 4

4. A particle is moving with simple harmonic motion. It achieves a maximum speed of 15 m s^{-1} and a maximum acceleration of magnitude 60 m s^{-2} .
Find its velocity 2 seconds after passing through the centre of the oscillation and interpret your answer. 5

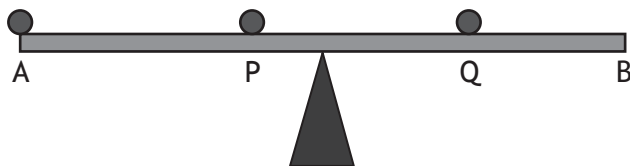
5. Find the solution of the second order differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$$

- given that $y = 1$ and $\frac{dy}{dx} = 3$, when $x = 0$. 5

[Turn over

6. A uniform rod AB, of length 8 metres and mass 30 kg, is placed on a support at its centre.



Masses of 10 kg, 5 kg and 12 kg are placed at the points A, P and Q respectively, as shown in the diagram, where AP is 3 metres and QB is 2 metres.

- (a) Calculate the magnitude and direction of the turning effect on the rod about its support. 3

The support is repositioned so that the rod is in equilibrium.

- (b) Determine how far the support was moved from its original position. 3

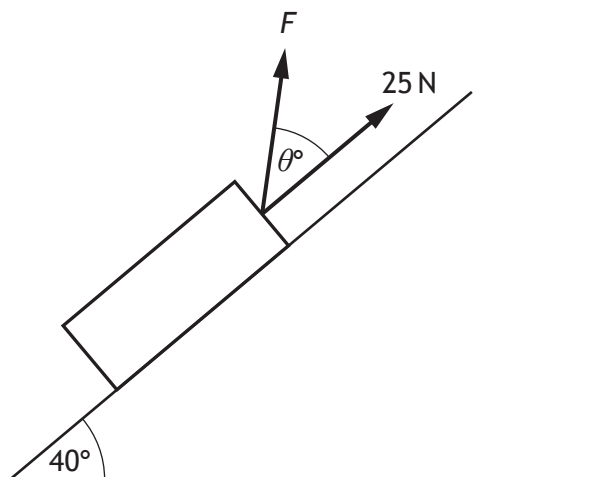
7. A function, f , is defined on a suitable domain by $f(t) = \ln(\sec 2t + \tan 2t)$. Differentiate $f(t)$ and simplify your answer. 4

8. A particle, starting from rest, moves with an acceleration of $2t\sqrt{2t+1} \text{ ms}^{-2}$, where t is the time in seconds.

Use integration by parts, or otherwise, to determine the velocity of the particle after 4 seconds. 5

9. A box of mass 5 kg rests on a smooth surface that is inclined at an angle of 40° to the horizontal.

Two external forces are applied to hold the box in equilibrium. These are a force of magnitude 25 newtons that is parallel to the slope, and a force of magnitude F newtons at an angle θ° to the slope as shown in the diagram.



The normal reaction force between the box and the slope is of magnitude 30 newtons. Calculate the angle θ° , and the magnitude of the force F .

5

10. A curve is defined implicitly by $3y + x^2 e^{2y} = 9$, $x > 0$.

Find the gradient of the tangent to the curve when $y = 0$.

4

11. A particle of mass 2 kg is projected from an origin along the x -axis with an initial velocity of 5 m s^{-1} .

A variable force of magnitude $0.2v^2$ newtons acts in the opposite direction to the initial motion of the particle, where v is the velocity of the particle in metres per second.

Find an expression for the velocity of the particle in terms of its displacement, x metres.

Give your answer in the form $v = pe^{qx}$, where p and q are constants.

5

[Turn over

12. A section of motorcycle track consists of a circular bend of radius r metres banked at an angle θ° to the horizontal.

The minimum speed that a motorcyclist can ride around the bend without slipping is

$$v = \frac{\sqrt{gr}}{10} \text{ m s}^{-1}.$$

- (a) Show that the coefficient of friction μ can be expressed as

$$\mu = \frac{100 \tan \theta^\circ - 1}{\tan \theta^\circ + 100}.$$

5

The circular bend is banked at an angle of 25° to the horizontal and has a radius of 80 m.

A motorcyclist approaches the bend at a speed of 28 m s^{-1} .

- (b) Determine whether the motorcyclist can travel around the bend at this speed without slipping.

4

On another occasion, a different motorcyclist approaches the same bend at the same speed of 28 m s^{-1} , but slides off the track.

- (c) State one possible reason for this outcome and justify your answer.

1

13. A body of mass m kilograms is projected with speed $V \text{ m s}^{-1}$ up a rough plane inclined at an angle θ to the horizontal.

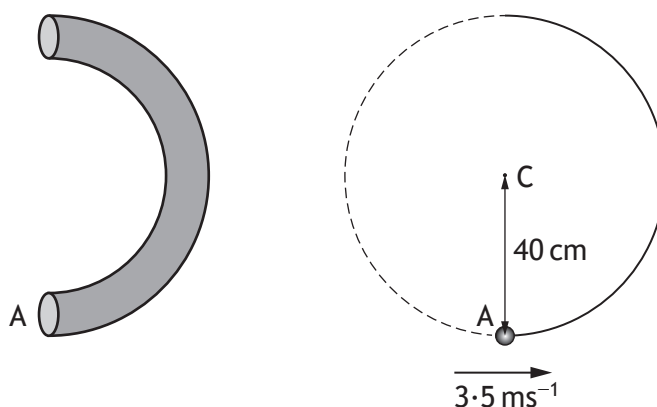
The body comes to rest after travelling a distance of s metres up the slope.

The coefficient of friction between the body and the slope is μ .

(a) Show that $s = \frac{V^2}{2g(\mu \cos \theta + \sin \theta)}$. 4

(b) Given that the work done against friction is equal to $\frac{1}{8}mV^2$ joules, find an expression for μ in terms of θ . 3

14. A vertical semicircle of radius 40 cm is formed from a length of smooth pipe as shown in the diagram. A ball is projected with a speed of 3.5 m s^{-1} from A, the bottom of the semicircle.



The centre of the circular path is the point C and the ball comes to instantaneous rest at a point P.

(a) Find the size of angle PCA. 4

The ball is projected from A again with an initial speed of u metres per second.

(b) Determine the restriction on u required for the ball to exit at the top of the pipe. 3

(c) Given that the ball acts as a particle, state another assumption that has been made about the ball in your solution. 1

[Turn over for next question

15. A ball is kicked from floor level at an angle of θ with initial speed $u \text{ m s}^{-1}$ in a room of height 3 metres.

(a) Show that, if the ball does not hit the ceiling, $\sin \theta < \frac{\sqrt{6g}}{u}$. 3

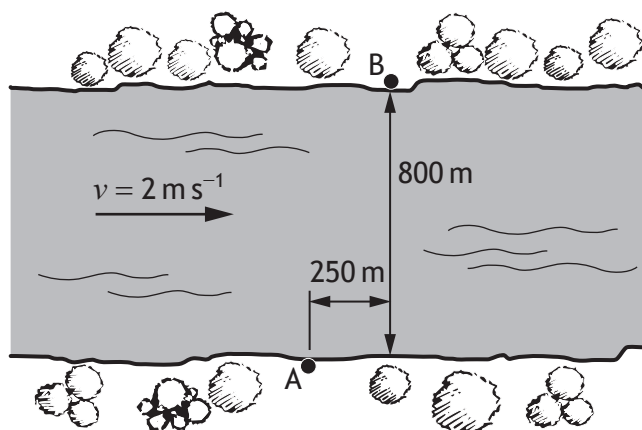
(b) The ball just touches the smooth ceiling at the highest point of its trajectory.

(i) Show that the range of the ball is $12\sqrt{\frac{u^2 - 6g}{6g}}$ metres. 5

(ii) State the constraint that must be placed on the initial speed of the ball in this case. 1

16. A rower is crossing a river that is 800 metres wide. They set off from point A and need to reach point B as quickly as possible. B is 250 metres downstream.

They row at a speed of 4 m s^{-1} in still water, and the river current flows at 2 m s^{-1} .



(a) Find at what angle to the bank the rower needs to steer. 4

After rowing for one minute the rower gets tired and immediately reduces speed to 3 m s^{-1} , adjusting steering to maintain the same course.

(b) (i) Find how far they are from B at this time. 3

(ii) Calculate the total time it takes the rower to reach point B. 3

17. (a) Find $\int e^t \sec^2(e^t) dt$ 1

(b) Given your answer describes the displacement of a particle, explain why the particle will never come to rest. 2

[END OF QUESTION PAPER]