X204/13/01

NATIONAL QUALIFICATIONS 2015

THURSDAY, 30 APRIL
1.00 PM – 4.00 PM

APPLIED MATHEMATICS
ADVANCED HIGHER
Mechanics

Read carefully

1. Calculators may be used in this paper.
2. Candidates should answer all questions.

   Section A assesses the Units Mechanics 1 and 2
   Section B assesses the Unit Mathematics for Applied Mathematics

3. **Full credit will be given only where the solution contains appropriate working.**

Section A (Mechanics 1 and 2)

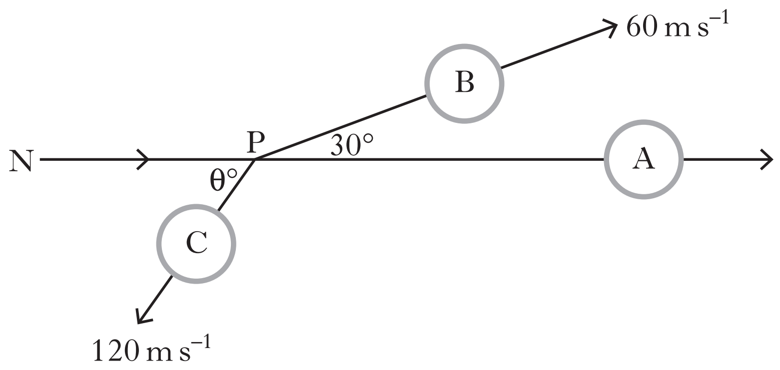
Marks

Answer all the questions

Candidates should observe that g m s^{-2} denotes the magnitude of the acceleration due to gravity.

Where appropriate, take its magnitude to be 9·8 m s^{-2}.

A1. A shell of mass 20 kg is travelling in a horizontal plane along the line NP at 100 m s^{-1}. At P it breaks into 3 pieces A, B and C of masses 12 kg, 6 kg and 2 kg respectively. These pieces instantaneously travel as shown in the diagram.



Find the speed of A, and the size of angle θ°. **4**

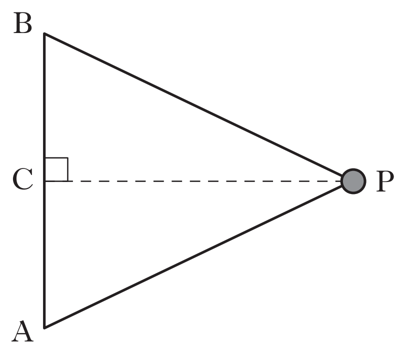
A2. An automated train is programmed to move from rest under constant acceleration to a maximum speed of 20 m s^{-1} in a distance of 300 m. It is brought to rest under uniform deceleration in 15 seconds. Two stations are 5 kilometres apart and the train is programmed to stop at each station.

Find the time taken to travel between the two stations. **5**

A3. A catapult consists of an elastic string of natural length 40 cm and modulus of elasticity 25 N. One end of the string is fixed to A and the other to B, with AB of length 20 centimetres.

A particle of mass 20 g is held in the middle of the string and the string pulled back to P.

It is held at rest with PC of length 24 centimetres. A, B, P and C all lie on the same horizontal plane. The particle is then released.

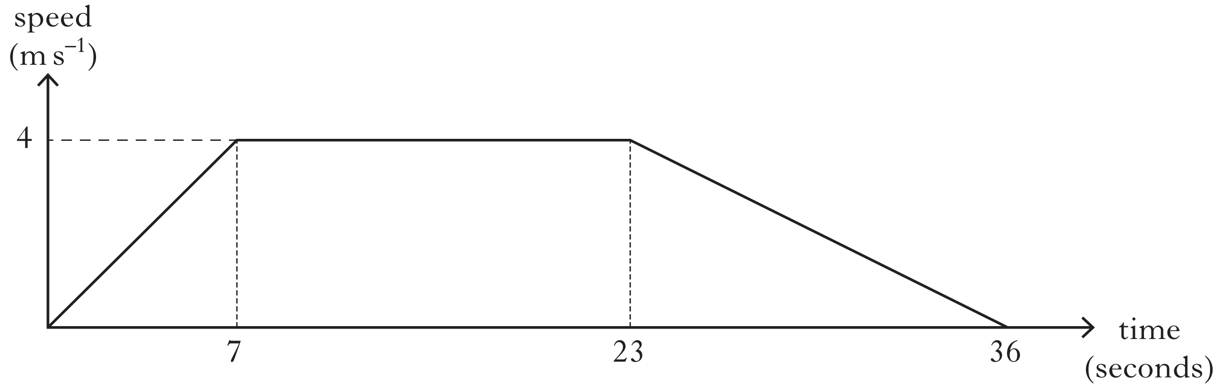


Find the speed with which it passes through C. **5**

Marks

A4. A lift and its occupants are limited to a mass of 1800 kg. It is to be drawn up and down a lift-shaft by an engine, using an inextensible cable.

A speed/time graph for the lift's ascent is shown.



(a) Find the power of the engine when the lift is fully loaded and travelling with a constant speed. **2**

(b) Find the maximum power generated during the ascent. **2**

(c) Find the vertical distance travelled by the lift. **2**

A5. Two moons Bart and Casper complete circular orbits about a planet.

Bart orbits with velocity v_B $m\,s^{-1}$ and radius r metres.

Casper orbits with velocity v_C $m\,s^{-1}$ and radius $2r$ metres.

(a) Show that $v_B = \sqrt{2}\,v_c$ and find the relationship between the angular velocities of the moons. **4**

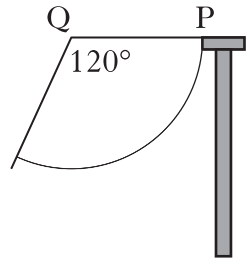
(b) If Bart has a period of n days, find the period of Casper's orbit. **2**

You should assume that Newton's Inverse Square Law of Gravitation applies.

[Turn over

Marks

A6. An acrobat of mass 60 kilograms starts her routine sitting on a platform P.



She is holding a rope of length 4 metres which is attached to a fixed support Q on the same horizontal level as P.

With the rope taut and horizontal, she drops off the platform and swings in a circular arc.

When she has swung through 120° she lets go of the rope.

(*a*) Show that her speed on the point of release is approximately $8{\cdot}24\,\text{m s}^{-1}$ and find the tension in the rope at that time **4**

After letting go of the rope, she moves freely under gravity.

(*b*) Find how far she rises before starting her descent. **2**

A7. At 3pm, a yacht is travelling with velocity $(4\mathbf{i} + 20\mathbf{j})$ km hr^{-1} while a trawler, positioned due north of the yacht, is travelling with a velocity $(-3\mathbf{i} - 4\mathbf{j})$ km hr^{-1}.

If the closest they get to each other in the subsequent motion is 4·2 kilometres, find the time to the nearest minute when they are closest and how far apart they were originally. **6**

A8. A particle P of mass 3 kg falls from rest under gravity. Throughout its motion, it experiences a resistance of $0{\cdot}25v^2$ newtons per unit mass, where v is the speed in metres per second at time t.

(*a*) Calculate the distance travelled by the body in reaching a speed of $5\,\text{m s}^{-1}$. **5**

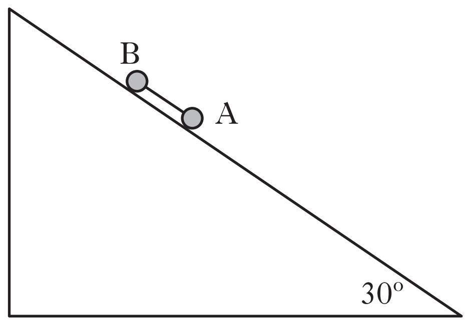
A second particle Q, of mass 5 kg, starts from rest at the origin and moves in a horizontal straight line with acceleration $2t\,\mathbf{i}\,\text{m s}^{-2}$, where t is the time in seconds from the start of the motion and $\mathbf{i}$ is the unit vector in the direction of motion.

(*b*) Given that the work done by the total force acting on Q during the first a seconds is equal to that done by the particle P in reaching its speed of $5\,\text{m s}^{-1}$, calculate the value of a. **5**

Marks

A9. Two particles A and B of mass 30 grams and 20 grams respectively are held on an inclined plane.

A is below B and 2 metres from the bottom of the slope.



The particles are joined by a taut, light inextensible string of length 25 cm which is parallel to the line of greatest slope of the plane.

The plane is inclined at 30° to the horizontal. Contact between particle A and the plane is smooth while the coefficient of friction between B and the plane is 0·5. The particles are released from rest and move down the slope.

(*a*) (i) Find the tension in the string and the acceleration of the particles. **4**

(ii) Calculate the speed of B at the instant when it reaches the original position of A. **2**

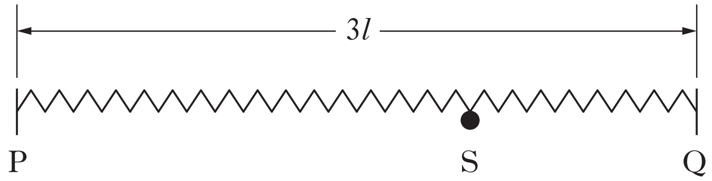
When B reaches the original position of A the string snaps and the two particles travel independently down the slope.

(*b*) Calculate the time interval between A and B reaching the bottom of the slope. **4**

[Turn over

Marks

A10. Two light elastic springs, each of natural length l metres are attached to a particle S of mass m kg. The particle initially lies in equilibrium on a smooth horizontal table, with the springs attached to two fixed points P and Q, a distance of $3l$ metres apart as shown in the diagram.



The spring attached at P has modulus of elasticity mg newtons and the spring attached at Q has modulus of elasticity $3\,mg$ newtons.

(*a*) Show that, in equilibrium, the distance PS = $\frac{7}{4}l$. **4**

S is then moved along the line PQ so that PS = $\frac{3}{4}l$ and is then released from rest at this point.

(*b*) (i) By finding in terms of l the extensions in PS and QS when the particle is x metres from its equilibrium position, show that the particle subsequently moves with simple harmonic motion. **4**

(ii) Show that the maximum velocity of the particle can be written in the form $k\sqrt{gl}$ and state the value of k. **2**

[*END OF SECTION A*]

Section B (Mathematics for Applied Mathematics)

Marks

Answer all the questions

B1. Given that $y = e^{5x} \tan 2x$, find $\dfrac{dy}{dx}$. **3**

B2. (*a*) Given matrix $A = \begin{pmatrix} 3 & -5 \\ 1 & -1 \end{pmatrix}$, find A^2 and show that the inverse of A^2 exists. **2**

(*b*) Hence, or otherwise, find matrix B such that $A^2B = \begin{pmatrix} 4 & 6 \\ 2 & -2 \end{pmatrix}$. **3**

B3. A curve is defined by

$$y = \frac{\sin x}{2 - \cos x} \text{ for } 0 \le x \le \pi.$$

Find the exact values of the coordinates of the stationary point of this curve. **5**

B4. Express $\log_a 2 + \log_a 4 + \log_a 8$ in the form $p\log_a 2$, where p is a constant. **1**

Hence evaluate $\sum_{r=1}^{100} \log_a 2^r$, giving your answer in the form $q\log_a 2$, where q is a constant. **3**

B5. Find the general solution, in the form $y = f(x)$, of the differential equation

$$\frac{1}{\cos x}\frac{dy}{dx} + y\tan x = \tan x,\ 0 < x < \frac{\pi}{2}$$ **6**

B6. (*a*) Express $\dfrac{1}{1 - y^2}$ in partial fractions. **3**

(*b*) Use the substitution $u = \sqrt{1 - x}$ to obtain $\displaystyle\int \frac{dx}{x\sqrt{1 - x}}$, $0 < x < 1$. **6**

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