# X204/13/01

NATIONAL
QUALIFICATIONS
2014

THURSDAY, 8 MAY
1.00 PM – 4.00 PM

# APPLIED MATHEMATICS
## ADVANCED HIGHER
### Mechanics

**Read carefully**

1. Calculators may be used in this paper.
2. Candidates should answer all questions.

    Section A assesses the Units Mechanics 1 and 2
    Section B assesses the Unit Mathematics for Applied Mathematics

3. **Full credit will be given only where the solution contains appropriate working.**

## Section A (Mechanics 1 and 2)

*Marks*

**Answer all the questions**

**Candidates should observe that $g$ m s$^{-2}$ denotes the magnitude of the acceleration due to gravity.**

**Where appropriate, take its magnitude to be 9·8 m s$^{-2}$.**

**A1.** Two particles, $P$ and $Q$, of masses 2 kg and $m$ kg respectively, initially lie at rest in a straight line on a smooth horizontal surface. Particle $P$ is acted on by a constant force of 3 N for 4 seconds, causing it to accelerate towards $Q$. When $P$ collides with $Q$ the particles coalesce and begin to move with speed 3·75 m s$^{-1}$.

Find the value of $m$, the mass of particle $Q$. **4**

**A2.** An object oscillates on the $x$-axis about the origin, $O$, with simple harmonic motion. The period of the oscillation is $\frac{14\pi}{5}$ seconds, and when the object is at point $K$, 1·2 metres from the origin, it is moving away from the origin with a velocity of 2·5 m s$^{-1}$.

Find the amplitude of the motion and the time it takes for the particle to travel from $O$ to $K$. **4**

**A3.** A car of mass 700 kg is at rest on a horizontal plane when it is acted on by a force $(3000 - 15x)\mathbf{i}$ N where $x$ is the displacement in metres. There is a constant frictional force of 500 N opposing motion.

When $x = 200$ find

(i) the total work done on the car; **2**

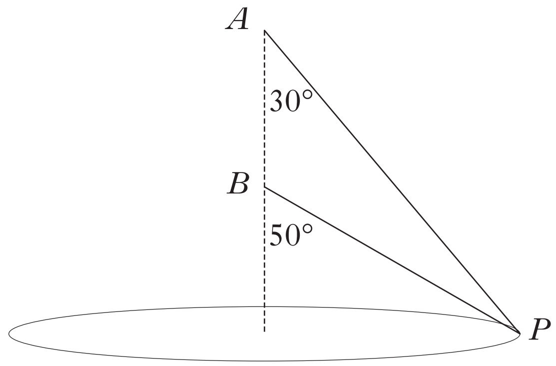
(ii) the speed of the car. **2**

*Marks*

**A4.** Two light inextensible strings each have one end attached to a particle $P$ of mass 2 kg. The other ends of the strings are attached to fixed points $A$ and $B$ where $A$ is vertically above $B$.

The particle moves with a constant speed in a horizontal circle whose centre is 30 cm below $B$. When the strings are inclined at 30° and 50° to the vertical, both strings are taut with the tension in $AP$ twice that in $BP$.

Find the linear speed of the particle. **6**




**A5.** A body of mass $M$ kg is moving upwards on a rough plane inclined at an angle $\theta°$ to the horizontal, where $\tan\theta = \frac{3}{4}$. As it passes through point $A$ it has a speed of $u$ m s$^{-1}$. It momentarily comes to rest at $B$ and subsequently passes through a point $C$ when moving down the slope with speed $2u$ m s$^{-1}$.

Given that the coefficient of friction between the mass and the slope is $\frac{1}{4}$, show that $AC = \dfrac{35u^2}{8g}$. **6**

**A6.** At 3pm, a patrol vessel travelling at a constant speed of 20 km h$^{-1}$ sights a ship 15 km away to the North East. The ship is travelling due North at a constant speed of 10 km h$^{-1}$.

Find the bearing on which the patrol vessel should travel to intercept the ship and the time at which this will occur. **6**

**A7.** A lift is initially at rest at the top of a building. At the instant the lift begins to descend with acceleration $\frac{1}{9}g$ m s$^{-2}$, a man in the lift releases a ball from a height of 1 metre above the lift floor by throwing it vertically upwards with a speed of 3·5 m s$^{-1}$.

The man then allows the ball to fall to the floor.

Assuming that the ball does not strike the ceiling of the lift, find the time taken for the ball to hit the floor from its moment of release. **7**

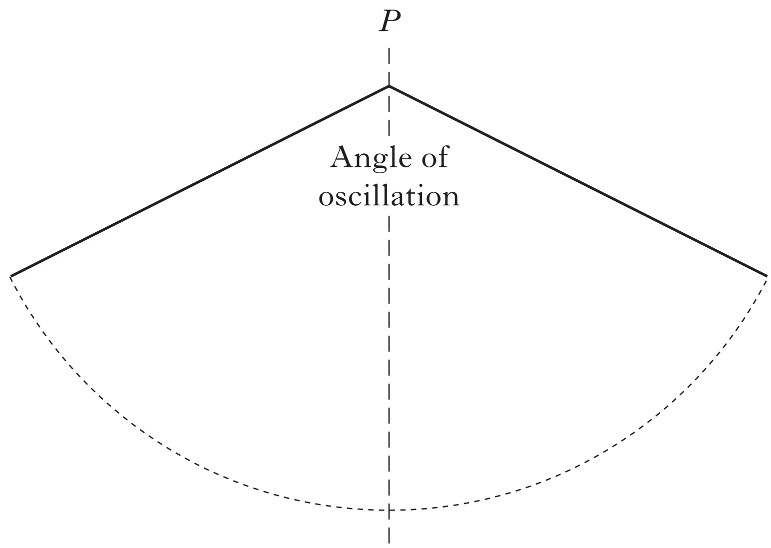
**[Turn over**

*Marks*

**A8.** A light rod $PQ$ of length 0·9 m has a particle of mass 3 kg attached at $Q$. The rod is free to rotate in a vertical plane about $P$. When $Q$ is vertically below $P$ the mass is given a horizontal velocity $u\,\mathrm{m\,s^{-1}}$ causing the rod to move in the vertical plane.

($a$) Show that, for the rod to complete a full circle, $u > \sqrt{\dfrac{18g}{5}}$. **3**

On another occasion, the horizontal velocity given to the mass is $4\,\mathrm{m\,s^{-1}}$ and the rod oscillates.




($b$) Find the angle of oscillation and the greatest tension in the rod during its motion. **7**

*Marks*

**A9.** An athlete competing in a long jump event accelerates from rest down the runway towards the take-off board.

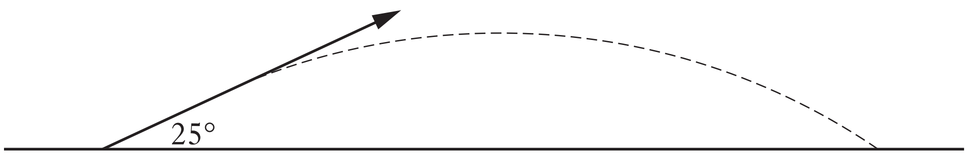
(*a*) On his first attempt, his acceleration can be modelled by the function

$$a = 13\left(\frac{3}{8} - \frac{t}{16}\right)\text{m s}^{-2} \text{ for } 0 \le t \le \frac{5}{2}$$

where $t$ is measured in seconds from the instant he sets off.

(i) Find the speed reached when $t = \frac{5}{2}$. **3**

(ii) The athlete maintains this speed to the take-off board. Using this as the speed of projection, calculate the horizontal distance that the athlete would jump if his take-off angle is 25° to the horizontal, as shown below. **3**




(*b*) On his second attempt, the athlete jumps a distance of 7·51 metres, having reached a take-off speed of $10{\cdot}2\ \text{m s}^{-1}$.

(i) Calculate the possible angles of projection for this jump. **3**

(ii) By considering the maximum height achieved in each case, show clearly which of these angles of projection represents his take-off angle. **2**

**[Turn over**

*Marks*

**A10.** A mass $m$ kilograms hangs at the end of a light inextensible string and is raised vertically by an engine working at a constant rate of $kmg$ watts.

(*a*) Show that the equation of the subsequent motion of the mass is $v^2\dfrac{dv}{dx}=(k-v)g$

where $v$ is the upward velocity of the mass and $x$ is its vertical displacement. **2**

(*b*) Initially the mass is at rest and when it is at height $h$ metres above its initial position it has a speed of $u\,\mathrm{m\,s^{-1}}$.

Show that $gh = k^2\ln\left|\dfrac{k}{k-u}\right|-ku-\dfrac{1}{2}u^2$ .

[You may assume that $\dfrac{a^2}{b-a}=-a-b+\dfrac{b^2}{b-a}$ ]

By finding the increase in the total energy of the mass, deduce that the time taken for the motion is $\dfrac{k}{g}\ln\left|\dfrac{k}{k-u}\right|-\dfrac{u}{g}$ . **8**

[*END OF SECTION A*]

**[Turn over for Section B on *Pages eight* and *nine***

## Section B (Mathematics for Applied Mathematics)

*Marks*

**Answer all the questions**

**B1.** Find the gradient of the tangent to the curve

$$y = 2x\sqrt{x-1}$$

at the point where $x = 10$. **4**

**B2.** Matrices are given as

$$A = \begin{pmatrix} 1 & 3 & 4 \\ k & 0 & -1 \\ 5 & 3 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & -10 & 2 \\ -3 & 9 & 0 \\ 0 & -2 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 3 & 2 & -6 \\ 1 & 1 & -2 \\ 2 & 2 & -1 \end{pmatrix}.$$

(*a*) Calculate $A + B$. **1**

(*b*) Find the determinant of $A$. **2**

(*c*) Calculate $BC$. **1**

(*d*) Describe the relationship between $B$ and $C$. **2**

**B3.** Find the exact value of $\int_0^{2\pi} x \sin 3x \, dx$. **5**

**B4.** Evaluate $\sum_{r=1}^{80} 3r^2$. **2**

**B5.** (*a*) Write down and simplify the binomial expansion of $(e^x + 2)^4$. **3**

(*b*) Hence obtain $\int (e^x + 2)^4 \, dx$. **2**

*Marks*

**B6.** A flu-like virus starts to spread through the 20 000 inhabitants of Dumbarton.

The situation can be modelled by the differential equation

$$\frac{dN}{dt} = \frac{N(20000 - N)}{10000},$$

where $N$ is the number of people infected after $t$ days and $0<N<20\,000$.

(*a*) How many people are infected when the infection is spreading most rapidly? **1**

(*b*) Express $\dfrac{10000}{N(20000-N)}$ in partial fractions and show that

$$\ln \frac{N}{(20000-N)} = 2t + C, \text{ for some constant } C.$$ **5**

Initially there were 100 people infected.

(*c*) Show that $N = \dfrac{20000\,e^{2t}}{199 + e^{2t}}$. **4**

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