National
Qualifications
2026

2026 Mathematics of Mechanics

Advanced Higher

Question Paper Finalised Marking Instructions

These marking instructions have been prepared by examination teams for use by Qualifications Scotland appointed markers when marking external course assessments.

Qualifications Scotland
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General marking principles for Advanced Higher Mathematics of Mechanics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

The marking instructions for each question are generally in two sections:

generic scheme — this indicates why each mark is awarded
illustrative scheme — this covers methods which are commonly seen throughout the marking

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example $6 \times 6 = 12$, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.

This is no longer a solution of a quadratic equation, so the mark is not awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$x = 1$$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$(x-3)(x-1) = 0$$
$$x = 1 \text{ or } 3$$

(i) Horizontal/vertical marking

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	○5	○6
○5	$x = 2$	$x = -4$
○6	$y = 5$	$y = -7$

Horizontal: ○5 $x = 2$ and $x = -4$ ○6 $y = 5$ and $y = -7$

Vertical: ○5 $x = 2$ and $y = 5$ ○6 $x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0{\cdot}3}$ must be simplified to 50 $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 144 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $\left(x^3 + 2x^2 + 3x + 2\right)(2x + 1)$ written as

  $\left(x^3 + 2x^2 + 3x + 2\right) \times 2x + 1$

  $= 2x^4 + 5x^3 + 8x^2 + 7x + 2$

  gains full credit

- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Marking Instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.	(a)		$\bullet^1$ use conservation of momentum $\bullet^2$ determine velocity	$\bullet^1$ $9\times 8+3\times(-4)=12v$ stated or implied by $\bullet^2$ $\bullet^2$ $5\ \text{ms}^{-1}$	**2**
Notes:					
Commonly Observed Responses: Where candidates omit direction, do not award $\bullet^1$. $\bullet^2$ is still available for an answer of $7\ \text{ms}^{-1}$ $\bullet^1$ $9\times 8+3\times 4=12v$ do not award $\bullet^1$ $\bullet^2$ $7\ \text{ms}^{-1}$					
	(b)		$\bullet^3$ calculate distance after $t=3$	$\bullet^3$ $7.5\ \text{m}$	**1**
Notes: 1. From an incorrect value of $7\ \text{ms}^{-1}$ in part (a), $\bullet^3$ is available for an answer of $10.5\ \text{m}$.					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
2.			$\bullet^1$ state expression $\bullet^2$ form equation and obtain one of A, B or C $\bullet^3$ state expression	$\bullet^1$ $\frac{A}{x+1}+\frac{Bx+C}{x^2+3x+5}$ $\bullet^2$ $5x^2+4x+2$ $=A(x^2+3x+5)+(Bx+C)(x+1)$ **AND** any one from $A=1$, $B=4$, $C=-3$ $\bullet^3$ $\frac{1}{x+1}+\frac{4x-3}{x^2+3x+5}$	**3**
Notes: 1. Where an incorrect template has been used, award 0/3. 2. For a correct answer without working, award full marks. 3. At $\bullet^3$ accept values explicitly stated for A, B and C provided the template is written at $\bullet^1$					
Commonly Observed Responses:					
3.			$\bullet^1$ calculate ω $\bullet^2$ find maximum acceleration $\bullet^3$ determine maximum force	$\bullet^1$ 8 $\bullet^2$ $8^2\times1.5$ $\bullet^3$ 24 N MUST BE POSITIVE – question asks for magnitude	**3**
Notes: 1. If the acceleration is calculated using an invalid strategy, or acceleration = 1.5 is used, then $\bullet^3$ is not available. 2. If an attempt has been made to calculate the acceleration using incorrect SHM equations, $\bullet^3$ is still available. 3. Candidate may use $x=a\sin(\omega t)$ with $x=a=1.5$ and $t=\frac{\pi}{16}$ for $\bullet^1$					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
4.			•[1] obtain equations for position of ball and fielder at time t •[2] calculate time of interception •[3] calculate distance travelled by the fielder	•[1] $r_b(t)=\begin{pmatrix}6t-2.5\\-4t+20\end{pmatrix}$ $r_f(t)=\begin{pmatrix}5t\\4t\end{pmatrix}$ •[2] 2.5 seconds •[3] $\frac{5\sqrt{41}}{2}$ or 16.0 metres	**3**
Alternative method					
			•[1] finds equation of straight lines •[2] solves simultaneously •[3] calculate distance travelled by the fielder	•[1] $y=\frac{4}{5}x$ and $y=\frac{-4}{6}x+\frac{55}{3}$ •[2] eg $x=12.5$ or $y=10$ •[3] $\frac{5\sqrt{41}}{2}$ or 16.0 metres	

Notes:

1. Accept 16 metres

Commonly Observed Responses:

COR 1
Award 3/3.

•[1] ${}_b r_f(t)=\begin{pmatrix}t-2.5\\-8t+20\end{pmatrix}$

•[2] $\frac{d}{dt}\left|{}_b r_f(t)\right|^2=(t-2.5)+-8\times2(-8t+20)=0$ ➔ 2.5 seconds

•[3] $\frac{5\sqrt{41}}{2}$ or 16.0 metres

Question			Generic scheme	Illustrative scheme	Max mark
5.	(a)		$\bullet^1$ set up integral $\bullet^2$ determine velocity as a function of t	$\bullet^1$ $v = \int 3t^2 dt$ stated or implied by $\bullet^2$ $\bullet^2$ $v = t^3 + 1$	**2**
Notes:					
Commonly Observed Responses: Where a candidate uses SUVAT eg $v = 3t^3 + 1$, award 0/2.					
	(b)		$\bullet^3$ calculate speed when $t = 2$ $\bullet^4$ calculate work done	$\bullet^3$ $v = 9\ \text{ms}^{-1}$ $\bullet^4$ 160 J	**2**
Notes: 1. For any use of SUVAT, $\bullet^4$ is unavailable.					
Commonly Observed Responses: **COR 1** Use of $v = 3t^3 + 1$ from part (a): $\bullet^3$ $v = 25\ \text{ms}^{-1}$ $\bullet^4$ 1248 J award 2/2 **COR 2** Award 0/2. $\bullet^3$ $s = \int t^3 + 1 dt$ ➔ 6 m do not award $\bullet^3$ $\bullet^4$ 288 J do not award $\bullet^4$					
Alternative Method					
			$\bullet^3$ substitute force given in question and velocity from part (a) $\bullet^4$ calculate work done	$\bullet^3$ $\int 12t^2(t^3 + 1) dt$ $\bullet^4$ 160 J	**2**
Notes:					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
6.			$\bullet^1$ evidence of use of quotient rule with denominator and one term correct in the numerator	$\bullet^1$ $\dfrac{3e^{3x}\ln 2x - \ldots}{(\ln 2x)^2}$ **OR** $\dfrac{\ldots - \dfrac{1}{2x}\times 2\times e^{3x}}{(\ln 2x)^2}$	**2**
			$\bullet^2$ complete differentiation	$\bullet^2$ $\dfrac{3e^{3x}\ln 2x - \dfrac{1}{x}e^{3x}}{(\ln 2x)^2}$ or equivalent	
Notes:					
Commonly Observed Responses: For candidates that use the product rule: $\bullet^1$ $\dfrac{3e^{3x}}{\ln 2x} + \ldots$ $\bullet^2$ $\dfrac{3e^{3x}}{\ln 2x} - \dfrac{e^{3x}}{x(\ln 2x)^2}$					

Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)		$\bullet^1$ set up equation for the x and y coordinate of the centre of mass relative to A	$\bullet^1$ $\begin{pmatrix}20\\25\end{pmatrix}\times 2000mg+\begin{pmatrix}70\\3\end{pmatrix}\times 360mg=\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}\times 2360mg$	**3**
			$\bullet^2$ determine x-coordinate of the centre of mass	$\bullet^2$ $\bar{x}=27.6\,\text{cm}$	
			$\bullet^3$ determine y-coordinate of the centre of mass	$\bullet^3$ $\bar{y}=21.6\,\text{cm}$	

Notes:

1. Disregard the omission of m and/or g at $\bullet^1$
2. Candidates may work throughout in either cm or m, but if a combination of units is used then units must be explicitly stated at $\bullet^2$ and $\bullet^3$

Commonly Observed Responses:

COR 1

Shape	Area	Centre of mass
Rectangle 1	2000	$(20,25)$
Rectangle 2	360	(70, 3)
Lamina	2360	$(\bar{x},\bar{y})$

COR 2

Candidate calculates centre of mass for rectangle 2 incorrectly, lose $\bullet^1$

Shape	Area	Centre of mass
Rectangle 1	2000	$(20,25)$
Rectangle 2	360	(30, 3)
Lamina	2360	$(\bar{x},\bar{y})$

Centre of mass (21.5, 21.6) for mark $\bullet^2$ and $\bullet^3$

COR 3

Candidate takes moments about B, lose $\bullet^1$

Shape	Area	Centre of mass
Rectangle 1	2000	$(80,25)$
Rectangle 2	360	(30, 3)
Lamina	2360	$(\bar{x},\bar{y})$

For (±72.4, 21.6) award mark $\bullet^2$ and $\bullet^3$

Question			Generic scheme	Illustrative scheme	Max mark
7.	(b)		$\bullet^4$ use moment = force x perpendicular distance $\bullet^5$ solve to find F	$\bullet^4$ $F \times 100 = 9g \times 27.6$ $\bullet^5$ 24.36...N	**2**

Notes:

1. When using rounded values, $\bullet^5$ is only available to candidates who demonstrate the need for a force which is greater than or equal to the necessary minimum.
   DO NOT ACCEPT 24 N or 24.3 N.
2. $\bullet^5$ can only be awarded from a calculation involving moments.
3. For candidates working in metres, $\bullet^4$ is only available for explicitly including $F \times 1$, but $\bullet^5$ is available.

Commonly Observed Responses:

COR 1
If $\overline{x} = 21.5$, $F =$ 18.96..N

COR 2
If $\overline{x} = 72.4$, $F =$ 63.85..N

Question			Generic scheme	Illustrative scheme	Max mark
8.	(a)		•[1] start to calculate angle •[2] calculate required angle •[3] calculate bearing	•[1] eg $\frac{\sin x^\circ}{40}=\frac{\sin 35^\circ}{420}$ •[2] 3.13° •[3] 072°	**3**
Alternative Method					
			•[1] uses resultant vector •[2] eliminates velocity •[3] calculate bearing	•[1] eg $\begin{pmatrix}420\cos\theta\\420\sin\theta\end{pmatrix}+\begin{pmatrix}40\cos 20\\-40\sin 20\end{pmatrix}=\begin{pmatrix}v\cos 15\\v\sin 15\end{pmatrix}$ •[2] eg $434.8...\cos(\theta+75)=-23.75...$ •[3] 072°	**3**
Notes: 1. Accept 72°					
Commonly Observed Responses: **COR 1** If 420 is used as the resultant velocity, the vector method is eased, but results in the correct bearing. •[1] eg $\begin{pmatrix}x\\y\end{pmatrix}+\begin{pmatrix}40\cos 20\\-40\sin 20\end{pmatrix}=\begin{pmatrix}420\cos 15\\420\sin 15\end{pmatrix}$ do not award •[1] •[2] eg $\begin{pmatrix}368.10...\\122.38...\end{pmatrix}$ award •[2] •[3] 072° award •[3]					

Question			Generic scheme	Illustrative scheme	Max mark
8	(b)		$\bullet^4$ calculate the resultant velocity $\bullet^5$ calculate the time taken	$\bullet^4$ $452\,\text{kmh}^{-1}$ $\bullet^5$ 1.11 hours or 1hour and 6 minutes	**2**
Alternative Method					
			$\bullet^4$ cosine rule in t $\bullet^5$ calculate the time taken	$\bullet^4$ $420^2 = 40^2 + \left(\frac{500}{t}\right)^2 - 2\times 40\times\left(\frac{500}{t}\right)\cos 35°$ $\bullet^5$ 1.1058 hours or 1hour and 6 minutes	

Notes:

1. Accept 1 h 7 minutes
2. For the award of $\bullet^5$, some demonstration of calculating a resultant velocity must appear at $\bullet^4$. For $\frac{500}{420}$, award 0/2.
3. Candidates who find the corresponding distance to 420 can also be given full credit, see COR 2.
4. Use of basic Pythagoras or right angled trigonometry is not sufficient and considered eased therefore mark 4 and 5 are unavailable.

Commonly Observed Responses:

COR 1

420 used as the resultant velocity in part (a).

$\bullet^4$ $v = 387\,\text{kmh}^{-1}$ award $\bullet^4$

$\bullet^5$ 1.288… hours (1 hour 17 minutes) award $\bullet^5$

COR 2

$\bullet^4$ $s = \frac{500\sin 35°}{\sin 142°} = 465.82\,\text{km}$ award $\bullet^4$

$\bullet^5$ 1.1058 hours or 1hour and 6 minutes award $\bullet^5$

Question			Generic scheme	Illustrative scheme	Max mark
9.	(a)		$\bullet^1$ construct auxiliary equation $\bullet^2$ find general solution	$\bullet^1$ $4m^2-12m+9=0$ $\bullet^2$ $x=Ae^{1.5t}+Bte^{1.5t}$ or equivalent	**2**
Notes: 1. Where "$=0$" does not appear in the auxiliary equation, $\bullet^1$ is not available. 2. If a candidate uses 'y and x' instead of 'x and t', $\bullet^2$ is not available.					
Commonly Observed Responses:					
	(b)		$\bullet^3$ find derivative of general solution $\bullet^4$ find one constant $\bullet^5$ find second constant and state particular solution $\bullet^6$ solve particular solution equal to zero	$\bullet^3$ $\frac{dx}{dt}=1.5Ae^{1.5t}+Be^{1.5t}+1.5Bte^{1.5t}$ $\bullet^4$ $A=-3$ or $B=2$ $\bullet^5$ $x=2te^{1.5t}-3e^{1.5t}$ $\bullet^6$ 1.5 seconds	**4**
Notes: 1. Where a candidate does not give an expression for the derivative, $\bullet^3$ can be awarded for $3.5e^3=1.5e^3A+e^3B+3e^3B$ or equivalent. 2. Do not award $\bullet^5$ if "$x=$" does not appear at that stage. 3. If a candidate repeatedly uses 'y and x' instead of 'x and t', $\bullet^3$, $\bullet^4$ and $\bullet^5$ may be awarded, but $\bullet^6$ is unavailable. 4. For candidates whose general solution comes from an auxiliary equation with distinct roots, $\bullet^3$ is not available; however, marks $\bullet^4$, $\bullet^5$ and $\bullet^6$ are still available.					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
10.	(a)		•[1] find extension •[2] calculate tension in spring	•[1] 0.489… •[2] 22.8 N	**2**

Notes:

1. Candidates may work in centimetres.
2. Calculator in radian mode: mark •[1] is unavailable (for x = 0.339) but •[2] may still be awarded for T_s =15.8N.

Commonly Observed Responses:

	(b)		•[3] resolve horizontally •[4] calculate tension in rod	•[3] $T_S \cos 40° = T_R \cos 20°$ •[4] 18.6 N	**2**

Notes:

1. Calculator in radian mode: answer must be consistent with part (a), and so •[4] is only available for $T_R = -25.9$ N (do not award if positive value given).

Commonly Observed Responses:

COR 1
•[3] $T_S \sin 50° = T_R \cos 20°$
•[4] 18.6 N

COR 2
•[3] $T_S \sin 50° = T_R \sin 70°$
•[4] 18.6 N

COR 3
Incorrect resolving.
•[3] $T_S \sin 40° = T_R \sin 20°$ do not award •[3]
•[4] 42.8 N award •[4]

Question			Generic scheme	Illustrative scheme	Max mark
10.	(c)		•^{5} resolve vertically •^{6} calculate mass	•^{5} $T_S \cos 50° + T_R \cos 70° = mg$ •^{6} 2.1 kg	**2**

Notes: must be two forces

1. For the award of •^{6}, there must be evidence at •^{5} of two forces in addition to the weight.
2. Calculator in radian mode: •^{6} is available for 3.2 kg.

Commonly Observed Responses:

COR 1

•^{5} $T_S \sin 40° + T_R \sin 20° = mg$

•^{6} 2.1 kg

COR 2

•^{5} $T_S \cos 50° + T_R \sin 20° = mg$

•^{6} 2.1 kg

COR 3

Incorrect resolving, **but consistent with (b)** ie $T_S = 22.8,\ T_R = 42.8$

•^{5} $T_S \cos 40° + T_R \cos 20° = mg$

•^{6} 5.9 kg award 2/2

COR 4

Correct resolving in part (c) **following incorrect resolving in part (b)** ie $T_S = 22.8,\ T_R = 42.8$

•^{5} $T_S \cos 50° + T_R \cos 70° = mg$

•^{6} 2.98… kg award 2/2

Question			Generic scheme	Illustrative scheme	Max mark
11.	(a)		•1 find expression for force	•1 $\frac{1}{4}\left(12t^2+8t\right)$ or equivalent	**3**
			•2 state integral (with limits) to find impulse	•2 $\int_0^3 \frac{1}{4}\left(12t^2+8t\right)dt$	
			•3 integrate and evaluate impulse	•3 36 Ns	

Notes:

1. For a candidate who uses $I = Ft$, •1 is only available for the appearance of an expression for the force (see COR 1 and COR 2). •2 and •3 are unavailable.
2. Where a candidate indicates they are integrating a force (even if this is the acceleration), •2 and •3 are available. Otherwise see COR 4.
3. Disregard the omission of dt at •2.

Commonly Observed Responses:

COR 1

Use $I = Ft$

•1 $\frac{1}{4}\left(12t^2+8t\right)$

•2 $3t^3+2t^2$ do not award •2

•3 99 Ns do not award •3

COR 2

•1 $12(3)^2+8(3)=132$ do not award •1

•2 $\frac{1}{4}\times 132 = 33$ do not award •2

•3 99 Ns do not award •3

COR 3

Award 3/3.

•1 $\int a\ dt = 4t^3+4t^2+c$

•2 $\Delta p = mv-mu$
$=0.25(4\times 3^3+4\times 3^2+c)-0.25(4\times 0^3+4\times 0^2+c)$

•3 36 Ns

COR 4

•1 $\int_0^3 a\ dt$ do not award •1

•2 $\int_0^3 12t^2+8t\ dt$ do not award •2

•3 144 Ns award •3

Question			Generic scheme	Illustrative scheme	Max mark
11.	(b)		$\bullet^4$ equate impulse with change in momentum $\bullet^5$ calculate initial speed	$\bullet^4$ $mv - mu = 36$ $\bullet^5$ $u = 6\,\text{ms}^{-1}$	**2**

Notes:

Commonly Observed Responses:

COR 1

$$\int a\ dt = 4t^3 + 4t^2 + c$$

$\bullet^4$ $150 = 4(3)^3 + 4(3)^2 + c$

$c = 6$

COR 2

$\bullet^4$ $mv - mu = 99$

$\bullet^5$ $u = -246\,\text{ms}^{-1}$

Award a maximum of 1/2 for $\bullet^4$

COR 3

$\bullet^4$ $mv - mu = 144$ (using expression for acceleration instead of force in part (a))

$\bullet^5$ $u = -426\,\text{ms}^{-1}$

Award a maximum of 1/2 for $\bullet^4$

Question			Generic scheme	Illustrative scheme	Max mark
12.			$\bullet^1$ integrate to find "$uv-$"	$\bullet^1$ $\frac{x-3}{3}(2x-1)^{\frac{3}{2}}-\ldots$	**3**
			$\bullet^2$ differentiate to find "$\int u'v\,dx$"	$\bullet^2$ $\ldots\int\frac{1}{3}(2x-1)^{\frac{3}{2}}dx$	
			$\bullet^3$ obtain full solution	$\bullet^3$ $\frac{x-3}{3}(2x-1)^{\frac{3}{2}}-\frac{1}{15}(2x-1)^{\frac{5}{2}}+c$	

Notes:

1. Disregard the omission of the constant of integration.
2. Disregard the omission of dx at $\bullet^2$.
3. For a repeated error in the integration of $(ax+b)^n$, $\bullet^3$ may be awarded on follow through.

Commonly Observed Responses:

COR 1

Candidate chooses to differentiate $(2x-1)^{\frac{1}{2}}$ and integrate $x-3$. Do not award $\bullet^1$ but $\bullet^2$ is available. $\bullet^3$ is unavailable as the integration cannot be completed.

$\bullet^2$ $(2x-1)^{\frac{1}{2}}\left(\frac{x^2}{2}-3x\right)-\int\frac{\frac{x^2}{2}-3x}{(2x-1)^{\frac{1}{2}}}dx$

COR 2

Tabular method.

	D	I
+	$x-3$	$(2x-1)^{\frac{1}{2}}$
-	1	$\frac{(2x-1)^{\frac{3}{2}}}{3}$
+	0	$\frac{(2x-1)^{\frac{5}{2}}}{15}$

COR 3

By substitution (eg $u=2x-1$) maximum 2/3.

$\bullet^1$ $\frac{du}{dx}=2$ do not award $\bullet^1$

$\bullet^2$ $\int\frac{1}{2}\left(\frac{u+1}{2}-3\right)\sqrt{u}\,du$

$\bullet^3$ $\int\frac{u^{\frac{3}{2}}-5u^{\frac{1}{2}}}{4}du$ = $\frac{(2x-1)^{\frac{5}{2}}}{10}-\frac{5(2x-1)^{\frac{3}{2}}}{6}+c$

Question			Generic scheme	Illustrative scheme	Max mark
13.	(a)		•[1] resolve forces in equilibrium perpendicular to slope	•[1] $R = mg\cos 43° + 400\sin 43°$ **OR** 989.5N	**4**
			•[2] apply Newton's Second Law parallel to the slope	•[2] $750 + 400\cos 43° - mg\sin 43° - 0.35R = ma$	
			•[3] calculate acceleration	•[3] $0.278\ldots \text{ms}^{-2}$	
			•[4] find time taken	•[4] 7.58s	

Notes:

1. At •[3] $a = 0.279$ leads to 7.57 s at •[4].
2. Accept answers which round to 7.6 s
3. For •[3], at least three forces must appear on the left-hand side of the equation at •[2].
4. If $R = 100g\cos 43°$, mark •[1] is unavailable but marks •[2], •[3] and •[4] are available (see COR 2).
5. •[4] is only available where a positive acceleration appears at •[3].

Commonly Observed Responses:

COR 1
Resolve horizontally and vertically.
•[1] $750\cos 43° - \mu R\cos 43° - R\sin 43° + 400 = ma\cos 43°$
•[2] $750\sin 43 + R\cos 43° - mg - \mu R\sin 43° = ma\sin 43°$
•[3] $0.278\ldots \text{ms}^{-2}$
•[4] 7.58s

COR 2
•[1] $R = mg\cos 43° = 716.7\ldots$ do not award •[1]
•[2] $750 + 400\cos 43° - mg\sin 43° - 0.35R = ma$ award •[2]
•[3] $1.23\ldots \text{ms}^{-2}$ award •[3]
•[4] 3.601…s award •[4]

COR 3
Incorrect resolving (only marks •[2] and •[3] are available).
•[1] $R = mg\sin 43° + 400\cos 43° = 960.89\ldots$ do not award •[1]
•[2] $750 + 400\sin 43° - mg\cos 43° - 0.35R = ma$
•[3] $-0.302\ldots \text{ms}^{-2}$

Question			Generic scheme	Illustrative scheme	Max mark
13.	(b)		$\bullet^5$ resolve parallel and perpendicular to slope for new situation $\bullet^6$ combine equations by substituting for normal reaction force $\bullet^7$ solve to find new force	$\bullet^5$ $P\cos 43° + 0.35R = mg\sin 43°$ $R = mg\cos 43° + P\sin 43°$ $\bullet^6$ $P\cos 43° + 0.35\left(mg\cos 43° + P\sin 43°\right)$ $= mg\sin 43°$ $\bullet^7$ 430.4N	**3**

Notes:

1. If $R = mg\cos 43°$ is used, only mark $\bullet^6$ is available.
2. For candidates who use incorrect resolving, see COR 2.

Commonly Observed Responses:

COR 1
Resolve horizontally and vertically.

$\bullet^5$ $\mu R\cos 43° - R\sin 43° + P = 0$
$R\cos 43° - mg + \mu R\sin 43° = 0$

$\bullet^6$ $P = \dfrac{mg(\sin 43° - \mu\cos 43°)}{\cos 43° + \mu\sin 43°}$

$\bullet^7$ 430.4N

COR 2
Incorrect resolving following either correct or incorrect working in part (a):

$\bullet^5$ $P\sin 43° + 0.35R = mg\cos 43°$
$R = mg\sin 43° + P\cos 43°$

$\bullet^6$ $P\sin 43° + 0.35\left(mg\sin 43° + P\cos 43°\right) = mg\cos 43°$

$\bullet^7$ 514.7N

Marks $\bullet^5$, $\bullet^6$ and $\bullet^7$ are all available to a candidate who has incorrectly resolved in part (a) ie repeated error within a question (General Marking Principle (l)). Otherwise, $\bullet^5$ is not available (ie award a maximum of 2/3).

Question			Generic scheme	Illustrative scheme	Max mark
14.	(a)		$\bullet^1$ apply product or chain rule	$\bullet^1$ $2y\frac{dy}{dx}$ or $3y+3x\frac{dy}{dx}$	**5**
			$\bullet^2$ complete differentiation	$\bullet^2$ $6x+2y\frac{dy}{dx}+3y+3x\frac{dy}{dx}=0$	
			$\bullet^3$ express relationship between y and x at stationary points	$\bullet^3$ $y=-2x$ or equivalent	
			$\bullet^4$ substitute into equation of curve	$\bullet^4$ eg $3x^2+(-2x)^2+3x(-2x)-9=0$	
			$\bullet^5$ determine stationary points	$\bullet^5$ $(3,-6)$ and $(-3,6)$	

Notes:

1. Award $\bullet^2$ only where $\frac{dy}{dx}$ appears more than once after the candidate has completed their differentiation.
2. For the award of $\bullet^5$, there must be a linear relationship between y and x at $\bullet^3$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
14.	(b)		$\bullet^6$ apply product rule	$\bullet^6$ $2\left(\frac{dy}{dx}\right)^2+2y\frac{d^2y}{dx^2}$ **OR** $3\frac{dy}{dx}+3x\frac{d^2y}{dx^2}$	**3**
			$\bullet^7$ complete differentiation	$\bullet^7$ $6+2\left(\frac{dy}{dx}\right)^2+2y\frac{d^2y}{dx^2}+3\frac{dy}{dx}+3\frac{dy}{dx}+3x\frac{d^2y}{dx^2}=0$	
			$\bullet^8$ evaluate	$\bullet^8$ 2	

Notes:

Commonly Observed Responses:

Use of quotient rule from part (a)

$\bullet^6$ $\frac{\left(-6-3\frac{dy}{dx}\right)(3x+2y)-\ldots}{(3x+2y)^2}$ or $\frac{\ldots-(-6x-3y)\left(3+2\frac{dy}{dx}\right)}{(3x+2y)^2}$

$\bullet^7$ $\frac{\left(-6-3\frac{dy}{dx}\right)(3x+2y)-(-6x-3y)\left(3+2\frac{dy}{dx}\right)}{(3x+2y)^2}$

$\bullet^8$ 2

Question			Generic scheme	Illustrative scheme	Max mark
15.			$\bullet^1$ differentiate u	$\bullet^1$ $\dfrac{du}{dx}=6x$	**5**
			$\bullet^2$ determine new limits and begin to rewrite integral	$\bullet^2$ $\int_{1}^{16}\ldots\sqrt{u}\,du$	
			$\bullet^3$ complete substitution	$\bullet^3$ $\int_{\ldots}^{\ldots}\ldots(u+2)\,du$ stated or implied by $\bullet^4$	
			$\bullet^4$ find expression in integrable form	$\bullet^4$ $\int_{1}^{16}\left(u^{\frac{3}{2}}+2u^{\frac{1}{2}}\right)du$	
			$\bullet^5$ integrate and evaluate	$\bullet^5$ 493.2	

Notes:

1. Where a candidate attempts to process an integrand containing both u and x, only $\bullet^1$ and $\bullet^2$ are available.
2. Limits are not necessary for mark $\bullet^3$.
3. $\bullet^2$ can be awarded for resubstituting for x instead of evaluating new limits.

Question	Generic scheme	Illustrative scheme	Max mark
15.	**(continued)**		

Commonly Observed Responses:

COR 1

$\int \sqrt{u}\,(u+2)\,du$ award $\bullet^3$

$\left[\dfrac{2(3x^2-2)^{\frac{5}{2}}}{5}+\dfrac{4(3x^2-2)^{\frac{3}{2}}}{3}\right]_1^{\sqrt{6}}$ award $\bullet^2$ and $\bullet^4$

COR 2

Wrong limits in new integral and return to original variable.

$\int_1^{\sqrt{6}} \ldots\sqrt{u}\,du$ do not award $\bullet^2$

$\int_{\ldots}^{\ldots} \ldots(u+2)\,du$ award $\bullet^3$

$\left[\dfrac{2(3x^2-2)^{\frac{5}{2}}}{5}+\dfrac{4(3x^2-2)^{\frac{3}{2}}}{3}\right]_1^{\sqrt{6}}$ award $\bullet^4$

COR 3

Wrong limits in new integral and no return to original variable.

$\int_1^{\sqrt{6}} \ldots\sqrt{u}\,du$ do not award $\bullet^2$

$\int_{\ldots}^{\ldots} \ldots(u+2)\,du$ award $\bullet^3$

$\int_1^{\sqrt{6}}\left(u^{\frac{3}{2}}+2u^{\frac{1}{2}}\right)du$ award $\bullet^4$, but $\bullet^5$ is unavailable

COR 4

Use of integration by parts.

$\bullet^4\ \left[\dfrac{2}{3}(u+2)u^{\frac{3}{2}}\right]_1^{16}-\int_1^{16}\left(\dfrac{2}{3}u^{\frac{3}{2}}\right)du$

Question			Generic scheme	Illustrative scheme	Max mark
16.	(a)		•[1] calculate E_p •[2] calculate velocity from kinetic energy •[3] calculate time to hit wall using horizontal velocity •[4] calculate drop during this time and height of impact	•[1] 0.64 J •[2] $11.3\,ms^{-1}$ •[3] 0.44…seconds •[4] 1.04 m	**4**

Notes:

1. •[2] is only available for candidates attempting to use the formula for Elastic Potential energy.

Commonly Observed Responses:

COR 1
Wrong extension eg x = 0.06

•[1] 1.44 J — do not award •[1]
•[2] $16.97\,ms^{-1}$ — award •[2]
•[3] 0.294...seconds — award •[3]
•[4] 1.57 m — award •[4]

COR 2
Candidate uses gravitational potential energy, lose •[1], •[2] and •[4]
•[3] 0.798...seconds ¬ — award •[3]

COR 3
Wrong mass of 0.1 used.
•[1] 0.64 J
•[2] $3.577\ldots ms^{-1}$ — do not award •[2]
•[3] 1.39…seconds — award •[3]
•[4] unavailable

COR 4
Use of tension.
•[1] $T = 32N$ — do not award •[1]
•[2] $a = 3200\,ms^{-2}$ — do not award •[2]

$$s = ut + \frac{1}{2}at^2$$

•[3] eg $5 = 0 + \frac{1}{2} \times 3200 \times t^2$ — do not award •[3]

$t = 0.0559\ldots$

•[4] eg $h = 2 - 0.015\ldots$
$= 1.98\ldots$ — award •[4]

Question			Generic scheme	Illustrative scheme	Max mark
16.	(b)		$\bullet^5$ calculate loss of potential energy $\bullet^6$ calculate gain in kinetic energy and final speed	$\bullet^5$ 0.0939J $\bullet^6$ $12.1 ms^{-1}$	**2**
Alternative Method					
			$\bullet^5$ find vertical component of velocity $\bullet^6$ calculate speed using components	$\bullet^5$ 4.33… $\bullet^6$ $12.1 ms^{-1}$	
Notes:					
Commonly Observed Responses: **COR 1** Wrong time from wrong extension eg x = 0.06. $\bullet^5$ and $\bullet^6$ available. $\bullet^5$ 2.88… $\bullet^6$ $17.2 ms^{-1}$					

Question			Generic scheme	Illustrative scheme	Max mark
17.			•[1] state form of integrating factor •[2] find integrating factor •[3] rewrite as integral equation •[4] integrate right-hand side and find particular solution	•[1] $e^{\int \frac{-1}{x-1}dx}$ •[2] $\frac{1}{x-1}$ •[3] $\frac{y}{x-1}=\int (x-1)\,dx$ •[4] $y=\left(\frac{(x-1)^2}{2}-1\right)(x-1)$ **or** equivalent eg $y=\left(\frac{x^2}{2}-x-\frac{1}{2}\right)(x-1)$	**4**

Notes:

1. For any attempt to separate variables, award 0/4
2. Do not withhold •[1] or •[3] for the omission of 'dx' on the right-hand side
3. Where candidate writes $\int \frac{d}{dx}\ldots$ on left-hand side do not withhold •[3] provided the candidate later produces evidence that they have integrated with respect to x
4. If the constant of integration is omitted, •[4] is unavailable
5. If candidates do the expansion it may result in $y=\frac{1}{2}\left(x^3-3x^2+x+1\right)$

Commonly Observed Responses:

COR 1

•[1] $e^{\int \frac{1}{x-1}dx}$ do not award •[1]

•[2] $x-1$ award •[2]

•[3] $y(x-1)=\int (x-1)^3\,dx$ award •[3]

•[4] $y=\frac{(x-1)^3}{4}$ •[4] unavailable

Question			Generic scheme	Illustrative scheme	Max mark
18.	(a)		•[1] find height in terms of r,θ •[2] use conservation of energy •[3] substitute and re-arrange •[4] use $F=ma$ for centripetal force •[5] substitute for v^2 and complete	•[1] $h=r(1-\cos\theta)$ •[2] $\frac{1}{2}mu^2=\frac{1}{2}mv^2+mgh$ •[3] eg $v^2=u^2-2gr(1-\cos\theta)$ •[4] $T-mg\cos\theta=\frac{mv^2}{r}$ •[5] $T=\frac{m(u^2-2gr(1-\cos\theta))}{r}+mg\cos\theta$ leading to $T=\frac{mu^2}{r}+mg(3\cos\theta-2)$	**5**
Notes:					
Commonly Observed Responses:					
	(b)		•[6] recognise $T=0$ and cancel m •[7] substitute and solve for θ	•[6] $\frac{u^2}{r}+g(3\cos\theta-2)=0$ •[7] $\theta=103.9^o$	**2**
Notes: 1. •[7] is only available as a result of equating the tension to zero.					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
18.	(c)		•[8] find v^2 at $\theta = 90^o$ •[9] realise flight is vertical and use SUVAT for extra height gained •[10] calculate final height	•[8] $v^2 = 4.24$ •[9] eg $0 = 4.24 - 2gh$ •[10] 0.816 m	**3**

Notes:

1. For candidates who equate E_K at the bottom of the circle ($8m$) with E_P at the maximum height (mgh), full marks may be awarded.
2. Should evidence appear which indicates that a candidate has used Conservation of Energy between the bottom of the circle $\frac{1}{2}m \times 4^2$ and the maximum height mgH, then marks •[8], •[9] and •[10] are all available

In the case where energy is not used:

3. Should there be no evidence of calculating a new velocity at 90° and SUVAT is used with u = 4 ms^{-1}, then marks •[8] and •[9] should be withheld, but •[10] may be awarded for the appearance of 0.816... (or 40/49), whether or not the candidate then stops or, alternatively, adds 0.6 to their answer to give 1.416...
4. Should an attempt be made to calculate a new velocity at 90° but this velocity is incorrect (even if erroneously this is then given as 4 ms^{-1}), mark •[8] should be withheld but marks •[9] and •[10] remain available.

Commonly Observed Responses:

Alternative Method

	•[8] use conservation of energy to find v^2 at $\theta = 90^o$ •[9] use conservation of energy for body moving freely under gravity •[10] calculate final height	•[8] $v^2 = 4.24$ •[9] eg $mgh = \frac{1}{2}m \times 4.24$ •[10] 0.816 m	**3**

Question			Generic scheme	Illustrative scheme	Max mark
19.	(a)		•[1] obtain an expression for velocity or time of flight	•[1] $v = 40\cos 50°\mathbf{i} + (40\sin 50° - gt)\mathbf{j}$ **OR** $t = \frac{40\sin 50°}{g} \times 2$	**2**
			•[2] calculate the total time of flight	•[2] $t = 6.25\,\text{s}$	

Notes:

1. •[1] may be awarded for calculation of the time to maximum height ($t = 3.12\ldots\,\text{s}$) as long as subsequent working does not indicate that the candidate treats this as the time of flight.

Commonly Observed Responses:

COR 1
Award mark •[1]

$\ddot{x} = 0$	$\ddot{y} = -g$
$\dot{x} = 40\cos 50°$	$\dot{y} = -gt + 40\sin 50°$
$x = 40t\cos 50°$	$y = \frac{-gt^2}{2} + 40t\sin 50°$

	(b)		•[3] link velocity with angle of flight	•[3] eg $\tan 20° = \frac{v}{40\cos 50°}$	**3**
			•[4] find one value for t	•[4] eg 2.17 s	
			•[5] find both times	•[5] 2.17 and 4.08	

Notes:

Accept 1.91s i.e. the time difference.

Commonly Observed Responses:

COR 1
For evidence of using $40\sin 20°$ leading to 1.73 and 4.52, award 1/3 (ie •[5]).

COR 2

$\tan 20° = \frac{\dot{y}}{\dot{x}}$ award •[3]

COR 3
$v = 40\cos 50°\tan 20°$ use of SUVAT leads to s = 43.435… and t = 2.17 (via a quadratic in t $4.9t^2 - 40\sin 50°t + 43.435$).

Question			Generic scheme	Illustrative scheme	Max mark
20.	(a)		•[1] differentiate to find expression for velocity •[2] calculate any two points •[3] calculates third point	•[1] $v = 2t - 5$ •[2] eg $(2.5, 0)$ or $(0, -5)$ •[3] eg $(5.5, 6)$	**3**
Notes: 1. If candidate integrates, award 0/3					
Commonly Observed Responses:					
	(b)	(i)	•[4] describe motion	•[4] the car has changed direction. Must reference change in direction.	**1**
Notes: 1. Accept 'the car goes backwards towards A and then forwards towards B'. 2. Accept 'velocity towards A, then velocity towards B'. 3. Where a candidate produces an incorrect statement before, during or after a correct statement, •[4] is unavailable.					
Commonly Observed Responses:					
		(ii)	•[5] identifies displacement at $t = 5.5$ •[6] identifies initial displacement and subtracts	•[5] 6.75m •[6] 2.75m	**2**
Alternative method					
		(ii)	•[5] finds displacement during reversing using area under graph. •[6] finds displacement in the direction AB from area under graph and subtracts	•[5] 6.25m •[6] 2.75m	**2**
Notes:					
Commonly Observed Responses:					

Question			Generic scheme	Illustrative scheme	Max mark
20.	(c)		$\bullet^7$ adds to find total area	$\bullet^7$ 33.25 m	**1**
Notes:					
Commonly Observed Responses:					

[END OF MARKING INSTRUCTIONS]