AH National Qualifications 2026

X802/77/11

Mathematics of Mechanics

TUESDAY, 26 MAY

1:00 PM – 4:00 PM

Total marks — 100

Attempt ALL questions.

You may use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate. Any rounded answer should be accurate to an appropriate number of significant figures unless otherwise stated.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you used this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

You must leave your answer booklet on your desk; if you do not, you could lose all the marks for this paper.

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FORMULAE LIST

Newton's inverse square law of gravitation

$$F = \frac{GMm}{r^2}$$

Simple harmonic motion

$$v^2 = \omega^2(a^2 - x^2)$$

$$x = a\sin(\omega t + \alpha)$$

Centre of mass

Triangle: $\frac{2}{3}$ along median from vertex.

Semicircle: $\frac{4r}{3\pi}$ along the axis of symmetry from the diameter.

Coordinates of the centre of mass of a uniform lamina, area A square units, bounded by the equation $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$ is given by

$$\bar{x} = \frac{1}{A}\int_a^b xy\,dx \qquad \bar{y} = \frac{1}{A}\int_a^b \frac{1}{2}y^2\,dx$$

Standard derivatives	
$f(x)$	$f'(x)$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\ln x$	$\frac{1}{x}$
e^x	e^x

Standard integrals	
$f(x)$	$\int f(x)\,dx$
$\sec^2(ax)$	$\frac{1}{a}\tan(ax) + c$
$\frac{1}{x}$	$\ln\lvert x\rvert + c$
e^{ax}	$\frac{1}{a}e^{ax} + c$

Total marks — 100

Attempt ALL questions

Note that g m s^{-2} denotes the magnitude of the acceleration due to gravity. Where appropriate, take its magnitude to be 9.8 m s^{-2}.

1. Objects A and B are moving towards each other on a straight horizontal track.

A has mass 9 kg and is moving with speed 8 m s^{-1} when it collides with B.

B has mass 3 kg and is moving with speed 4 m s^{-1} in the opposite direction.

The objects combine and move together.

(a) Determine the speed immediately after the collision. **2**

Immediately after the collision, the combined objects start decelerating at a constant rate, until they come to rest after 3 seconds.

(b) Determine the distance travelled in this time. **1**

2. Express $\dfrac{5x^2+4x+2}{(x+1)(x^2+3x+5)}$ as a sum of partial fractions. **3**

3. A body of mass 0.25 kilograms moves horizontally with simple harmonic motion.

The period of the motion is $\dfrac{\pi}{4}$ seconds and the amplitude is 1.5 metres.

Find the magnitude of the greatest horizontal force the body experiences during its motion. **3**

4. A cricket ball is hit and rolls along the ground.

It moves from a position of $\begin{pmatrix} -2.5 \\ 20 \end{pmatrix}$ metres, relative to an origin, with a constant velocity of $\begin{pmatrix} 6 \\ -4 \end{pmatrix}$ m s^{-1}.

At the same time, a fielder sets off from the origin and runs with a constant velocity of $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$ m s^{-1} and intercepts the ball.

Calculate how far the fielder travels until the ball is intercepted. **3**

MARKS

5. A remote-control car has an acceleration of $3t^2$ m s^{-2}, where t seconds is the time from the start of the motion. It is initially moving at 1 m s^{-1}.

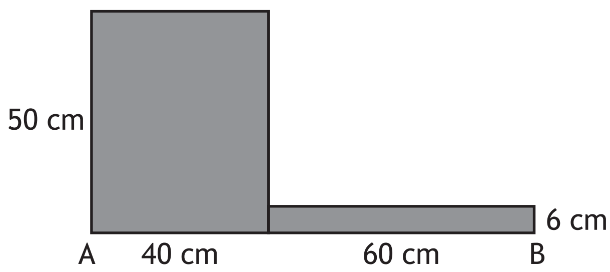
(a) Find an expression for the velocity, v, of the remote-control car as a function of t. **2**

The remote-control car has a mass of 4 kg.

(b) Given that the acceleration continues until $t = 2$, calculate the work done during this time. **2**

6. Given $f(x) = \dfrac{e^{3x}}{\ln 2x}$, $x > 0$, find $f'(x)$. **2**

7. Two uniform rectangles are joined together in a vertical plane to form one lamina of mass per unit area m kg.



(a) Determine the coordinates of the centre of mass relative to the point A. **3**

The total mass of the lamina is 9 kg. A vertical force of magnitude F is applied at the point B.

(b) By considering moments about A, find the value of F that will cause the lamina to begin to turn in an anti-clockwise direction. **2**

8. An aircraft leaves airport A and flies directly to airport B which is 500 km away on a bearing of 075°.

The aircraft has a speed of 420 km h^{-1} in still air and there is a wind blowing at a speed of 40 km h^{-1} on a bearing of 110°.

(a) Calculate the bearing the aircraft must take to fly from A to B. **3**

(b) Determine the time it takes for the aircraft to fly from A to B. **2**

9. The motion of a particle along a horizontal line over a short period of time, is described by the differential equation

$$4\frac{d^2x}{dt^2} - 12\frac{dx}{dt} + 9x = 0$$

where x represents the displacement in metres of the particle from the origin at time t seconds.

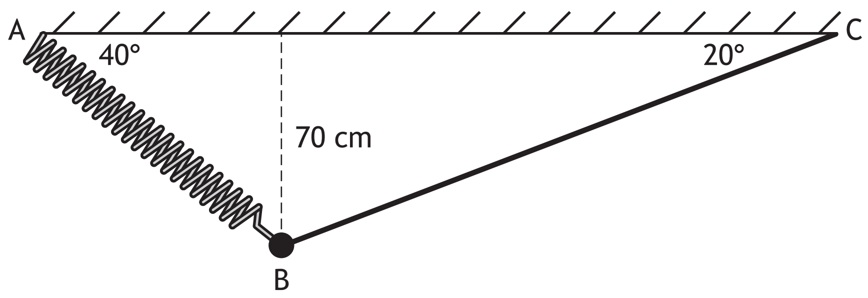
(a) Determine the general solution of this differential equation. **2**

It is observed that at 2 seconds the particle is e^3 metres from the origin and has velocity $3.5e^3$ metres per second.

(b) Calculate the time at which the particle passes through the origin. **4**

10. A particle of mass m kilograms is held in equilibrium by a spring and a light inextensible rod.

The particle is held at B, 70 cm below the horizontal AC, as shown in the diagram.



The spring has a natural length of 60 cm and a modulus of elasticity of 28 N.

Angle ACB is 20° and CAB is 40°.

(a) Calculate the tension in the spring. **2**

(b) Find the tension in the rod. **2**

(c) Determine the mass of the particle. **2**

[Turn over

MARKS

11. A particle of mass 0.25 kilograms is moving in a straight line. It is given an acceleration of $\left(12t^2+8t\right)$ m s^{-2} over 3 seconds by a force of magnitude F newtons.

(a) Calculate the impulse applied to the particle over this time. **3**

The force, F, acts in the same direction as the particle is moving.

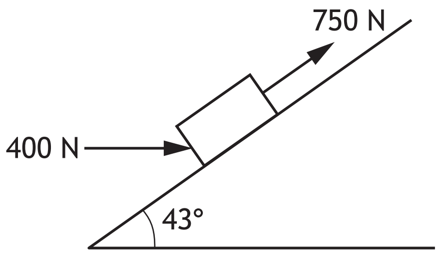
(b) Given that the speed of the particle after the application of the impulse is 150 m s^{-1}, calculate its speed before the impulse was applied. **2**

12. Use integration by parts to obtain $\int (x-3)\sqrt{2x-1}\,dx$. **3**

13. Two people move a crate of mass 100 kg up a slope angled at 43° to the horizontal. The crate is initially at rest.

One pulls the crate with a force of 750 newtons parallel to the slope.

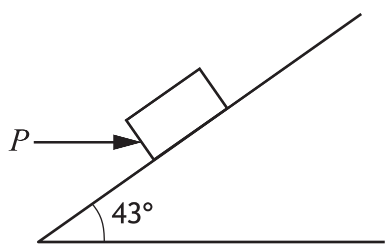
At the same time the other pushes the crate with a horizontal force of 400 newtons.



The coefficient of friction between the crate and the slope is 0.35.

(a) Calculate the time taken to move the crate 8 metres up the slope. **4**

The person pulling the crate lets go of it. The crate comes to rest, and the person pushing now applies a different horizontal force, P newtons, to stop the crate from slipping down the slope.



(b) Calculate the value of P. **3**

MARKS

14. A curve is defined by the equation $3x^2 + y^2 + 3xy - 9 = 0$.

(a) Use implicit differentiation to determine the coordinates of the two stationary points on the curve. **5**

(b) Evaluate the second derivative of this curve at the stationary point where $x > 0$. **3**

15. Use the substitution $u = 3x^2 - 2$ to evaluate $\int_1^{\sqrt{6}} 18x^3 \sqrt{3x^2 - 2}\, dx$. **5**

16. A horizontal spring of natural length 10 cm and modulus of elasticity 80 N is compressed to 6 cm.

The potential energy stored in the spring is used to launch a 10 gram projectile horizontally from a height of 2 metres. The projectile flies towards a vertical wall, which is 5 metres away horizontally.

(a) Find the height at which the projectile hits the wall. **4**

(b) Calculate the speed at which the projectile hits the wall. **2**

17. Find the particular solution of the differential equation

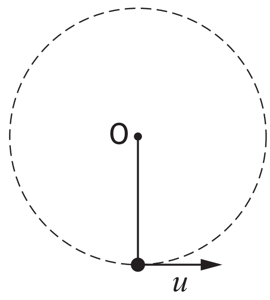
$$\frac{dy}{dx} - \frac{y}{x-1} = (x-1)^2$$

given that $y = 2$ when $x = 3$. Express your answer in the form $y = f(x)$. **4**

[Turn over

MARKS

18. A body of mass m kg is attached to a fixed point O by a light inextensible string. The body is given a horizontal velocity u m s^{-1} and then moves in a vertical circle of radius r metres.



θ is the angle between the string and the downward vertical through O. The string has tension T newtons.

(a) Use conservation of energy to show that $T = \dfrac{mu^2}{r} + mg\left(3\cos\theta - 2\right)$. **5**

(b) Given that the radius of the circle is 0.6 metres, and $u = 4$ m s^{-1}, find the angle θ at which the string goes slack. **2**

On another occasion the body is launched with $u = 4$ m s^{-1} and $r = 0.6$ m, but this time the string breaks when $\theta = 90°$ and the body then moves freely under gravity.

(c) Find the maximum height reached by the body, relative to the bottom of the circle. **3**

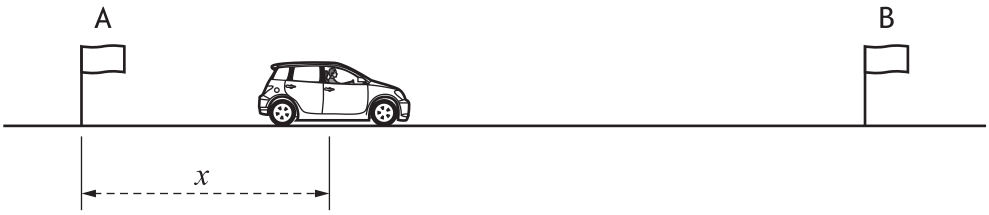
19. A particle is projected with a velocity of 40 m s^{-1} at an angle of 50° to the horizontal.

(a) Find the time of flight. **2**

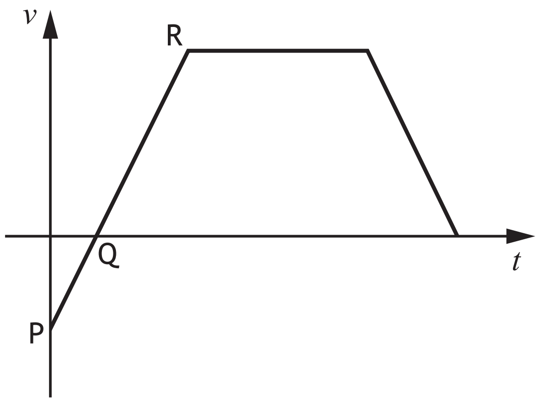
(b) Find the two times between which the angle of the velocity of the particle is less than 20° to the horizontal. **3**

20. A toy car moves on a straight horizontal track so that its displacement from flag A can initially be described by $x = t^2 - 5t + 4$ where x is measured in metres and t is the time in seconds.

It reaches its maximum velocity of 6 m s^{-1} at flag B and continues at this velocity. It then decelerates uniformly to rest.



(a) Calculate the coordinates of points P, Q and R on the velocity/time graph for this motion. **3**



(b) (i) Describe the motion of the car before and after point Q on the graph. **1**

(ii) Determine the distance between the starting point and flag B. **2**

The car travels a distance of 12 m at its maximum speed. It comes to rest after decelerating for 2 seconds.

(c) Calculate the total distance travelled. **1**

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