2011 Mathematics

Higher

Finalised Marking Instructions

General Comments

These marking instructions are for use with the 2011 Higher Mathematics Examination.

For each question the marking instructions are in two sections, namely **Illustrative Scheme** and **Generic Scheme**. The **Illustrative Scheme** covers methods which you will commonly see throughout your marking. The **Generic Scheme** indicates the rationale for which each mark is awarded. In general you should use the **Illustrative Scheme** and only use the **Generic Scheme** where a candidate has used a method not covered in the **Illustrative Scheme**.

All markers should apply the following general marking principles throughout their marking:

1 Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than deducted for what is wrong.

2 Award one mark for each •. There are no half marks.

3 Working subsequent to an error must be **followed through**, with possible full marks for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction of mark(s) should be made.

4 **Marking Symbols**
No comments, words or acronyms should be written on scripts. Please use the following and **nothing else**.

✓ A tick should be used where a piece of working is correct and gains a mark. You are not expected to tick every line of working but you must check through the whole of a response.

____ x Where a mark is lost, the error should be underlined in **red** at the point where it first occurs, and not at any subsequent stage of the working.

A cross-tick should be used to indicate "correct" working where a mark is awarded as a result of **follow through** from an error.

A double cross-tick should be used to indicate correct working which is irrelevant or insufficient to score any marks. This should also be used for working which has been **eased**.

~~~ A tilde should be used to indicate a minor transgression which is not being penalised, e.g. **bad form**.

This should be used where a candidate is given the **benefit of the doubt**.

^ A roof should be used to show that something is missing, such as a crucial step in the working or part of a solution.

These will help you to maintain consistency in your marking and will assist the examiners in the later stages of SQA procedures.
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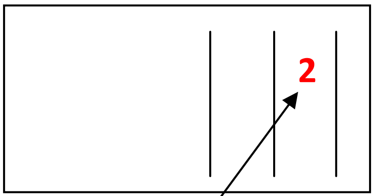
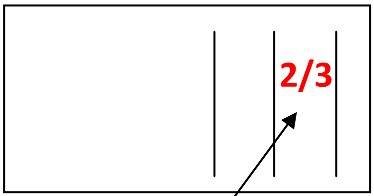
5 Regularly Occurring Responses (ROR) are shown on the marking scheme to help mark common solutions that are non-routine.

6 RORs may also be used as a guide in marking other non-routine candidate responses.

7 The mark for **each part** of a question should be entered in **red** in the **outer** right hand margin, opposite the end of the working concerned. The marks should correspond to those on the question paper and these marking instructions. Only the mark, **as a single number**, should be written.



Marks in this column - single numbers only



Do not record marks on scripts in this manner.

8 Where a candidate has scored zero for any question, or part of a question, 0 should be written in the right hand margin beside their answer.

9 Every page of a candidate's script should be checked for working. Unless blank, every page which is devoid of a marking symbol, should have a tick placed in the bottom right hand margin.

10 Where a solution is spread over several pages the marks should be recorded at the end of the solution. This should be indicated with a down arrow (↓), in the margin, at the earlier stages.

The examples below illustrate the use of the marking symbols .

Example 1

$y = x^3 - 6x^2$	$\bullet^1$	✓
$\frac{dy}{dx} = \underline{3x^2 - 12}$ ✓ $\bullet^1$ ✗ $\bullet^2$	$\bullet^2$	✗
$3x^2 - 12 = 0$ ✗ $\bullet^3$	$\bullet^3$	✗
$x = 2$ ^ $\bullet^4$	$\bullet^4$	^
$y = -16$ ✗ $\bullet^5$	$\bullet^5$	✗

Example 2

$A(4,4,0),\ B(2,2,6),\ C(2,2,0)$

$\overrightarrow{AB} = \underline{\mathbf{b} + \mathbf{a}} = \begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix}$ ✗ $\bullet^1$

$\overrightarrow{AC} = \begin{pmatrix} 6 \\ 6 \\ 0 \end{pmatrix}$ ✗ $\bullet^2$ (repeated error)

Example 3

$3\sin x - 5\cos x$

$k\sin x\cos a - \cos x\sin a$ ✓ $\bullet^1$

$k\cos a = 3,\ k\sin a = 5$ ✓ $\bullet^2$

Example 4

Find intersection of $x + 3y = 23$ and $y = 3x - 9$

$y - 3x = \underline{9}$

$3y + x = \underline{24}$ ✓ $\bullet^1$

Strategy mark awarded. (despite two errors)

$3y - 9x = 27$

$x = -\frac{3}{10}$ ✗ $\bullet^2$ The subsequent pd mark is lost (Note 12)

11 Where a transcription error (paper to script or within script) occurs, a mark is lost.
e.g.

This is a transcription error and so mark is lost.

Eased as not solution of a quadratic equation.

$$x^2 + 5x + 7 = 9x + 4$$
$$\underline{x - 4}x + 3 = 0$$
$$x = 1$$

Exceptionally this error is not treated as a transcription error as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$(x-3)(x-1) = 0$$
$$x = 1 \text{ or } 3$$

12 In general, as a consequence of an error perceived to be trivial, casual or insignificant, e.g. $6 \times 6 = 12$, candidates lose the opportunity of gaining the appropriate *ic* or *pd* mark.

13 A processing error made at a strategy mark stage is penalised at the next *pd* or *ic* mark available within that part of the question. The strategy mark may still be awarded.

14 As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Throughout this paper, unless specifically mentioned in the marking scheme, a correct answer with no working receives no credit.

15 Unless specifically mentioned in the marking scheme, do not penalise:

- Working subsequent to a **correct** answer;
- Correct working in the wrong part of a question;
- Legitimate variations in numerical answers, e.g. angles in degrees rounded to nearest degree;
- Omission of units;
- Bad form.

16 No piece of working should be ignored without careful checking – even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking scheme. Answers which are widely off-beam are unlikely to include anything of relevance, but candidates may still have the opportunity of gaining the odd mark or two, provided it satisfies the criteria for the marks.

17 In the exceptional circumstance where you are in doubt whether a mark should or should not be awarded, err on the generous side and award the mark.

18 Scored out or erased working which **has not been replaced** should be marked where still legible. However, if the scored out or erased working **has been replaced**, only the work which has not been scored out should be marked.

19 A valid approach, within Mathematical problem solving, is to try different strategies. Where this occurs, all working should be marked. The mark awarded to the candidate is from the *highest* scoring strategy. This is distinctly different from the candidate who gives two or more solutions to a question/part of a question, deliberately leaving all solutions, hoping to gain some benefit. All such contradictory responses should be marked and the *lowest* mark given.

20 It is of great importance that the utmost care should be exercised in adding up the marks.

The recommended procedure is as follows:

Step 1 Manually calculate the total from the candidate's script.

Step 2 Check this total using the grid issued with these marking instructions.

Step 3 Input the scores and obtain confirmation of your total from the EMC screen.
(This should highlight any discrepancies hitherto undiscovered.)

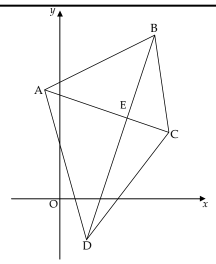
21 Place the candidate's script for Paper 2 inside the script for Paper 1 and write the candidate's total score (i.e. Paper 1 Section B + Paper 2) in the space provided on the front cover of the script for Paper 1.

22 In cases of difficulty, covered neither in detail nor in principle in these instructions, contact your Team Leader (TL) in the first instance. A referral to the Principal Assessor (PA) should only be made in consultation with your TL. Please see the General Marking Instructions for PA Referrals.

Question	Answer
1	C
2	B
3	D
4	D
5	A
6	C
7	D
8	A
9	B
10	D
11	D
12	C
13	C
14	B
15	B
16	A
17	A
18	C
19	C
20	D

Summary		
	A	4
	B	4
	C	6
	D	6

21 A quadrilateral has vertices A(−1, 8), B(7, 12), C(8, 5) and D(2, −3) as shown in the diagram.



(a) Find the equation of diagonal BD. **2**

(b) The equation of diagonal AC is $x+3y=23$.

Find the coordinates of E, the point of intersection of the diagonals. **3**

Generic Scheme	Illustrative Scheme
21 (a)	
$\bullet^1$ pd find gradient of BD	$\bullet^1$ $\frac{15}{5}$ or equivalent
$\bullet^2$ ic state equation of BD	$\bullet^2$ $y-(-3)=3(x-2)$ **or** $y-12=3(x-7)$ **2**

Notes

1. There is no need to simplify m_{BD} for $\bullet^1$; however, it must be simplified before $\bullet^2$ can be awarded.
2. If m_{BD} cannot be simplified, due to an error, then $\bullet^2$ is still available.
3. Candidates who determine the equation of AC lose $\bullet^1$ but may still gain $\bullet^2$.
4. Candidates lose $\bullet^1$ and $\bullet^2$ for the equation of any side of the quadrilateral.

Regularly occurring responses

Response 1

Using $y=mx+c$

$y=3x+c$ ✓ $\bullet^1$

$12=3\times7+c$ **or** $-3=3\times2+c$

$c=-9$ ✓ $\bullet^2$

2 marks out of 2

Response 2

$\left.\begin{array}{l} m_{AC}=-\frac{1}{3} \\ m_{BD}=3 \end{array}\right\}$ ✗ $\bullet^1$

Candidate has assumed diagonals are perpendicular - without evidence.

$y-(-3)=3(x-2)$ ✗ $\bullet^2$

1 mark out of 2

Generic Scheme	Illustrative Scheme
21 (b)	
$\bullet^3$ ss start solution of simultaneous equations	$\bullet^3$ e.g. $3x-y=9$ and $x+3y=23$ **or** $3x-9=-\frac{x}{3}+\frac{23}{3}$ **or** $x+3(3x-9)=23$
$\bullet^4$ pd solve for one variable	$\bullet^4$ $x=5$ **or** $y=6$
$\bullet^5$ pd solve for second variable	$\bullet^5$ $y=6$ **or** $x=5$ **3**

Notes

5. Candidates who find the equation of AC in (a), correctly or incorrectly, lose $\bullet^3$, $\bullet^4$ and $\bullet^5$ in (b).
6. Any other incorrect answer from (a) may still gain $\bullet^3$, $\bullet^4$ and $\bullet^5$ as follow through.

Regularly occurring responses

Response 3

$3x-y=3$ and $x+3y=23$ ✓ $\bullet^3$

Subsequent to gaining $\bullet^3$ an error was made in simplifying the equation in (a), but strategy mark still awarded in (b).

$x=3\cdot2$ ✗ $\bullet^4$

$y=6\cdot6$ ✓ $\bullet^5$

Error going from (a) to (b) is penalised at first pd (or ic) mark.

2 marks out of 3

21	(c)	(i) Find the equation of the perpendicular bisector of AB. (ii) Show that this line passes through E.	5

Generic Scheme			Illustrative Scheme	
21 (c)				
•6	ss	know and find midpoint of AB	•6	$(3,10)$
•7	pd	find gradient of AB	•7	$\frac{4}{8}$ or equivalent
•8	ic	interpret perpendicular gradient	•8	$-\frac{8}{4}$ or equivalent **stated, or implied by •9**
•9	ic	state equation of perp. bisector	•9	$y-10=-2(x-3)$ **but not** $y-6=-2(x-5)$
•10	ic	justification of point on line	•10	when $x=5,\ y=-2\times5+16=6$ **or** $2\times5+6-16=0$

5

Notes

7. Candidates who do not simplify the gradient in (a) and (c) should only be penalised once.
8. •9 is only available as a consequence of using a midpoint and perpendicular gradient.
9. Candidates who use $y-6=-2(x-5)$ at •9 stage, lose •9 and •10.
10. Candidates who show that the point of intersection of BD or AC **and** the perpendicular bisector is E gain •10.

Regularly occurring responses

Response 4

$m_{\text{PERP BISECTOR}}=-2$
$m_{\text{"ME"}}=\ldots=-2$
So perpendicular bisector goes through E ✗ •10

There must be reference to the midpoint being a common point to gain this mark.

Response 5

From (i) equation of perpendicular bisector is $y=-2x+16$, using $(3,\ 10)$.

Then in (ii) using $m=-2$ and E(5, 6) leads to $y=-2x+16$. Same equation so E lies on line. ✓ •10

Response 6

From (b) $E(3\cdot2,\ 6\cdot6)$
$x=3\cdot2,\ y=\ldots=9\cdot6$, so line does not pass through E. ✗ •10

Comment must be consistent with E from (b).

22 A function f is defined on the set of real numbers by $f(x) = (x-2)(x^2+1)$.

(a) Find where the graph of $y = f(x)$ cuts:

(i) the x-axis; (ii) the y-axis. **2**

(b) Find the coordinates of the stationary points on the curve with equation $y = f(x)$ and determine their nature. **8**

Generic Scheme			Illustrative Scheme		
22 (a)					
•¹	ic	interpret x intercept	•¹	$(2, 0)$	(minimum response "(i) 2")
•²	ic	interpret y intercept	•²	$(0, -2)$	(minimum response "(ii) -2")

2

Notes

1. Candidates who obtain extra x-axis intercepts lose •¹.
2. Candidates who obtain extra y-axis intercepts lose •².
3. Candidates who interchange intercepts can gain at most one mark.

Generic Scheme			Illustrative Scheme	
22 (b)				
•³	ic	write in differentiable form	•³	$x^3 - 2x^2 + x - 2$
•⁴	ss	know to and start to differentiate	•⁴	$3x^2 \ldots$ or $\ldots - 4x \ldots$
•⁵	pd	complete derivative and equate to 0	•⁵	$3x^2 - 4x + 1$ **and** $f'(x) = 0$ **or** $3x^2 - 4x + 1 = 0$
•⁶	pd	factorise derivative	•⁶	$(3x-1)(x-1)$
•⁷	pd	process for x	•⁷	$\frac{1}{3}$ and 1 ¦ $x = \frac{1}{3}$ and $y = -\frac{50}{27}$
•⁸	pd	evaluate y-coordinates	•⁸	$-\frac{50}{27}$ and -2 ¦ $x = 1$ and $y = -2$
•⁹	ic	justify nature of stationary points	•⁹	(table below)
•¹⁰	ic	interpret and state conclusions	•¹⁰	max, min

x	$\cdots$	$\frac{1}{3}$	$\cdots$	1	$\cdots$
$f'(x)$	+	0	−	0	+
		max		min	

Accept a valid expression in lieu of $f'(x)$.

8

This question may be marked vertically. The dotted rectangle shows what is required for •¹⁰.

Notes

4. •⁵ is only available if " $=0$ " appears at or before •⁶ stage.
5. •³, •⁴ and •⁵ are the only marks available to candidates who solve $3x^2 - 4x = -1$.
6. At •⁹ the nature can be determined using the second derivative.
7. •⁹ is only available if the nature table is consistent with the candidate's derivative.
8. •¹⁰ is awarded for correct interpretation of the candidate's nature table in words.

Regularly occurring responses

Response 1A

x	0	$\frac{1}{3}$	$\frac{1}{2}$	1	2
$\frac{dy}{dx}$	1	0	$-\frac{1}{4}$	0	5 ✓ •⁹
		max		min ✓ •¹⁰	

Response 1B

^	$\cdots$	$\frac{1}{3}$	$\cdots$	1	$\cdots$
$f'(x)$	+	0	−	0	+ ✗ •⁹
		max		min ✓ •¹⁰	

x missing

Response 1C

x	$\cdots$	$\frac{1}{3}$	$\cdots$	1	$\cdots$
slope	/	—	\	—	/

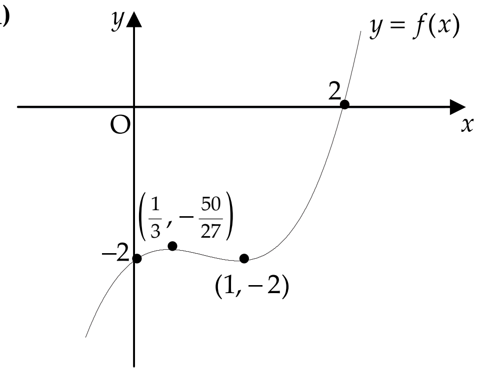
✗ •⁹
^ •¹⁰

signs or values are necessary

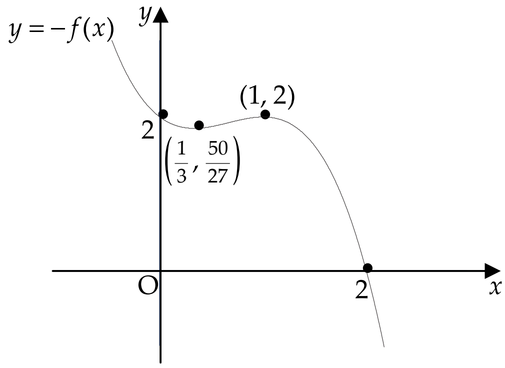
22	(c)	On separate diagrams sketch the graphs: (i) $y = f(x)$; (ii) $y = -f(x)$.	3

Generic Scheme | **Illustrative Scheme**

(c) (i)



(ii)



•[11]	ic	curve showing points from (a) and (b) without annotation	•[11]	sketch
•[12]	ic	**cubic** curve showing **all** intercepts and stationary points annotated	•[12]	sketch
•[13]	ic	curve from (i) reflected in $x-$axis	•[13]	reflected sketch

3

Notes

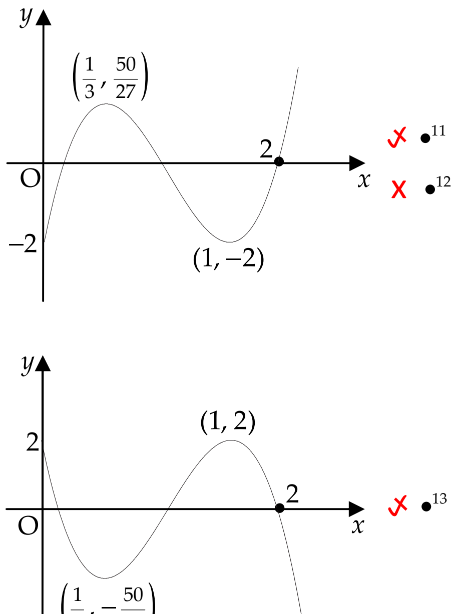
9. •[11] is for any curve consistent with all points found in (a) and (b). Ignore any extra critical points.

10. In (c)(ii), the minimum requirement is the curve from (c)(i) reflected in x-axis showing **at least one** x-intercept unchanged and **at least one** stationary point correctly annotated.

Regularly occurring responses

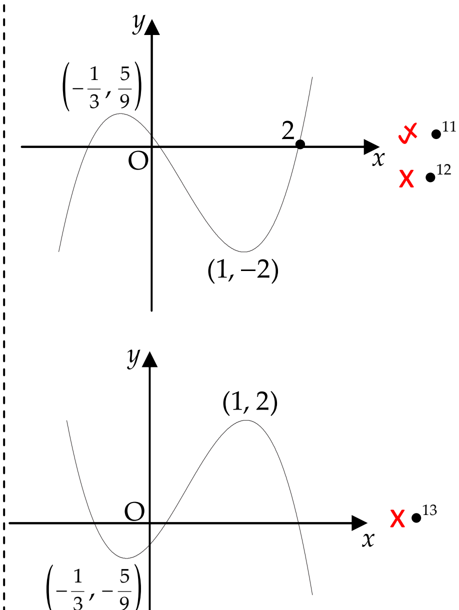
Follow through from candidate's work in (a) and (b).

Response 2



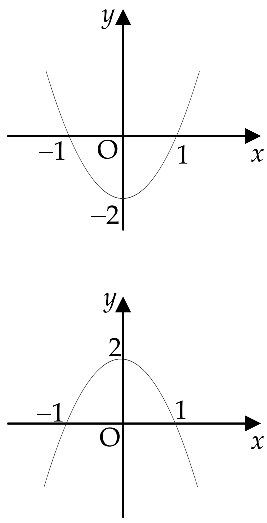
2 marks out of 3

Response 3



1 mark out of 3

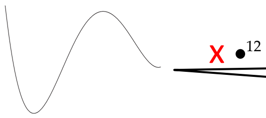
Response 4



0 marks out of 3

No marks available for a quadratic.

Response 5



Not a cubic due to apparent change of concavity.

23	(a)	Solve $\cos 2x^\circ - 3\cos x^\circ + 2 = 0$ for $0 \le x < 360$.	5

Generic Scheme	Illustrative Scheme
23 (a)	
	Method 1 : Using factorisation
•[1] ss know to use double angle formula	•[1] $2\cos^2 x^\circ - 1 \ldots$ **stated, or implied by** •[2]
•[2] ic express as a quadratic in $\cos x^\circ$	•[2] $2\cos^2 x^\circ - 3\cos x^\circ + 1$ } = 0 must appear at either
•[3] ss start to solve	•[3] $(2\cos x^\circ - 1)(\cos x^\circ - 1)$ } of these lines to gain •[2].
	Method 2 : Using quadratic formula
	•[1] $2\cos^2 x^\circ - 1 \ldots$
	•[2] $2\cos^2 x^\circ - 3\cos x^\circ + 1 = 0$ **stated explicitly**
	•[3] $\dfrac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 2 \times 1}}{2 \times 2}$
	In both methods :
•[4] pd reduce to equations in $\cos x^\circ$ only	•[4] $\cos x^\circ = \frac{1}{2}$ and $\cos x^\circ = 1$ — Candidates who include 360 lose •[5]
•[5] ic process solutions in given domain	•[5] 0, 60 and 300
	or
	•[4] $\cos x^\circ = 1$ and $x = 0$ — Candidates who include 360 lose •[4]
	•[5] $\cos x^\circ = \frac{1}{2}$ and $x = 60$ or 300

5

Notes

1. •[1] is not available for simply stating that $\cos 2A = 2\cos^2 A - 1$ with no further working.
2. In the event of $\cos^2 x - \sin^2 x$ or $1 - 2\sin^2 x$ being substituted for $\cos 2x$, •[1] cannot be awarded until the equation reduces to a quadratic in $\cos x$.
3. Substituting $\cos 2A = 2\cos^2 A - 1$ or $\cos 2a = 2\cos^2 a - 1$ etc. should be treated as bad form throughout.
4. Candidates may express the quadratic equation obtained at the •[2] stage in the form $2c^2 - 3c + 1$ or $2x^2 - 3x + 1$ etc. For candidates who do not solve a trigonometric quadratic equation at •[5], $\cos x$ must appear explicitly to gain •[4].
5. •[4] and •[5] are only available as a consequence of solving a quadratic equation.
6. Any attempt to solve $ax^2 + bx = c$ loses •[3], •[4] and •[5].
7. •[5] is not available to candidates who work in radian measure and do not convert their answers into degree measure.

Regularly occurring responses

Response 1

(Reading $\cos 2x^\circ$ as $\cos^2 x^\circ$)

$\cos^2 x^\circ - 3\cos x^\circ + 2 = 0$ ✗ •[1] ✗ •[2]

$(\cos x^\circ - 2)(\cos x^\circ - 1) = 0$ ✓ •[3]

$\cos x^\circ = 2$ or $\cos x^\circ = 1$ ✓ •[4]

no solution $x = 0$ ✓ •[5]

2 marks out of 5

Response 2A

(See note 6 above)

$2\cos^2 x^\circ - 1 - 3\cos x^\circ + 2 = 0$ ✓ •[1]

$2\cos^2 x^\circ - 3\cos x^\circ = -1$ ✗ •[2]

$\cos x^\circ(2\cos x^\circ - 3) = -1$ ✗ •[3]

$\cos x^\circ = -1$ or $\cos x^\circ = 1$ ✗ •[4]

$x = 180$ $x = 0$ ✗ •[5]

1 mark out of 5

Response 2B

(See note 6 above)

$2\cos^2 x^\circ - 1 - 3\cos x^\circ + 2 = 0$ ✓ •[1]

$2\cos^2 x^\circ - 3\cos x^\circ + 1 = 0$ ✓ •[2]

$2\cos^2 x^\circ - 3\cos x^\circ = -1$

$\cos x^\circ(2\cos x^\circ - 3) = -1$ ✗ •[3]

$\cos x^\circ = -1$ or $\cos x^\circ = 1$ ✗ •[4]

$x = 180$ $x = 0$ ✗ •[5]

2 marks out of 5

23 (b) Hence solve $\cos 4x^\circ - 3\cos 2x^\circ + 2 = 0$ for $0 \le x < 360$. **2**

Generic Scheme			Illustrative Scheme
23 (b)			
•6	ic	interpret relationship with (a)	•6 $2x = 0$ and 60 and 300
•7	ic	interpret periodicity	•7 0, 30, 150, 180, 210 and 330

2

Notes

8. Do not penalise the inclusion of 360 in (b).

9. Ignore extra answers, correct or incorrect, outside the given interval, but penalise incorrect answers within the interval.

10. Do not penalise candidates who use radians in (b) if they have already been penalised in (a).

11. Candidates who go back to 'first principles' for (b) can only gain •6 and •7 for a correct method leading to valid solutions as stated in the Illustrative Scheme.

Regularly occurring responses

Response 3A

From (a) $x = 0,\ 60,\ 300$

(b) $\cos 4x^\circ - 3\cos 2x^\circ + 2 = 0$

$2(\cos 2x^\circ - 3\cos x^\circ + 1) = 0$ ✗ •6

$x = 0,\ 30,\ 150,\ 180,\ 210,\ 330$ ✓ •7

1 mark out of 2

Response 3B

From (a) $x = 0,\ 60,\ 300$

(b) ^ •6

$x = 0,\ 30,\ 150,\ 180,\ 210,\ 330$ ✓ •7

1 mark out of 2

Response 4A

From (a) $x = 0,\ 60,\ 300$

(b) $x \div 2 = 0,\ 30,\ 150$ ^ •6 ✗ •7

0 marks out of 2

Response 4B

From (a) $x = 0,\ 60,\ 300$

(b) $x \div 2 = 0,\ 30,\ 150,\ 180,\ 210,\ 330$ ^ •6 ✓ •7

1 mark out of 2

Response 5

From (a) $x = 0,\ 60,\ 300$

(b) $\cos(2.2x^\circ) - 3\cos 2x^\circ + 2 = 0$ ✓ •6

$x = 0,\ 30,\ 150,\ 180,\ 210,\ 330$ ✓ •7

2 marks out of 2

Response 6

From (a) $x = 0,\ 60,\ 300$

(b) period $\div 2$ ✓ •6

so $x = 0,\ 30,\ 150,\ 180,\ 210,\ 330,\ 360,\ 570$ ✓ •7

2 marks out of 2

Response 7

From (a) $x = 0,\ 60,\ 300$

(b) $2x$ repeats every 180 ^ •6

$x = 0,\ 60,\ 300,\ 0+180,\ 60+180$

$= 0,\ 60,\ 180,\ 240,\ 300$ ✗ •7

0 marks out of 2

Response 8 (Wrong angles from (a))

e.g. $x = 0,\ 30,\ 330$

(b) $2x = 0,\ 30,\ 330$ ✓ •6

$x = 0,\ 15,\ 165,\ 180,\ 195,\ 345$ ✓ •7

2 marks out of 2

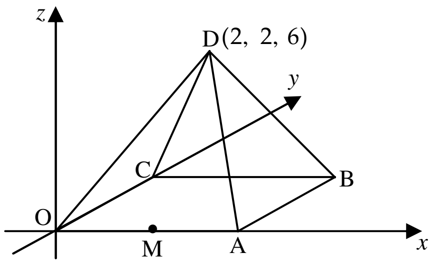
1 D,OABC is a square based pyramid as shown in the diagram below.

O is the origin, D is the point $(2,\ 2,\ 6)$ and $OA = 4$ units.

M is the mid-point of OA.

(a) State the coordinates of B. 1

(b) Express $\overrightarrow{DB}$ and $\overrightarrow{DM}$ in component form. 3



Throughout this question, coordinates written as components and vice versa are treated as bad form.

Generic Scheme			Illustrative Scheme
1 (a)			
•¹	ic	state coordinates of B	•¹ $(4,\ 4,\ 0)$
			1

Generic Scheme			Illustrative Scheme
1 (b)			
•²	pd	state components of $\overrightarrow{DB}$	•² $\begin{pmatrix}2\\2\\-6\end{pmatrix}$
•³	ic	state coordinates of M	•³ $(2,\ 0,\ 0)$ **stated, or implied by** •⁴
•⁴	pd	state components of $\overrightarrow{DM}$	•⁴ $\begin{pmatrix}0\\-2\\-6\end{pmatrix}$
			3

Regularly occurring responses

Response 1A (Transcription error for D)

$\overrightarrow{DB} = \begin{pmatrix}4\\4\\0\end{pmatrix} - \underline{\begin{pmatrix}2\\6\\6\end{pmatrix}} = \begin{pmatrix}2\\-2\\-6\end{pmatrix}$ ✗ •²

$\overrightarrow{DM} = \begin{pmatrix}2\\0\\0\end{pmatrix} - \begin{pmatrix}2\\6\\6\end{pmatrix} = \begin{pmatrix}0\\-6\\-6\end{pmatrix}$ ✓ •³ ✓̸ •⁴

2 marks out of 3

Response 1B (Transcription error for D)

$\overrightarrow{DB} = \begin{pmatrix}2\\-2\\-6\end{pmatrix}$ and $\overrightarrow{DM} = \begin{pmatrix}0\\-6\\-6\end{pmatrix}$ with no working.

0 marks out of 3

Response 2A

$\overrightarrow{DB} = \underline{\mathbf{d} + \mathbf{b}} = \begin{pmatrix}6\\6\\6\end{pmatrix}$ ✗ •²

$\overrightarrow{DM} = \mathbf{d} + \mathbf{m} = \begin{pmatrix}4\\2\\6\end{pmatrix}$ ✓ •³ ✓̸ •⁴

2 marks out of 3

Response 2B

$\overrightarrow{DB} = \begin{pmatrix}6\\6\\6\end{pmatrix}$ and $\overrightarrow{DM} = \begin{pmatrix}4\\2\\6\end{pmatrix}$ with no working.

0 marks out of 3

1	(c)	Find the size of angle BDM.	5

Generic Scheme			Illustrative Scheme	
1 (c)				
•5	ss	know to use scalar product	•5 $\cos B\hat{D}M = \dfrac{\overrightarrow{DB}\,.\,\overrightarrow{DM}}{\lvert\overrightarrow{DB}\rvert\lvert\overrightarrow{DM}\rvert}$	**stated, or implied by •9**
•6	pd	find scalar product	•6 $\overrightarrow{DB}\,.\,\overrightarrow{DM} = 32$	
•7	pd	find magnitude of a vector	•7 $\lvert\overrightarrow{DB}\rvert = \sqrt{44}$	
•8	pd	find magnitude of a vector	•8 $\lvert\overrightarrow{DM}\rvert = \sqrt{40}$	
•9	pd	evaluate angle BDM	•9 $40\cdot3^\circ$ or $0\cdot703$ rads	

5

Notes

1. •5 is not available to candidates who evaluate the wrong angle.
2. If candidates do not attempt •9, then •5 is only available if the formula quoted relates to the labelling in the question.
3. •9 should be awarded to any answer which rounds to 40° or $0\cdot7$ rads.
4. In the event that both magnitudes are equal **or** there is only one non-zero component, •8 is not available.

Regularly occurring responses

Response 3A

$\cos B\hat{O}M = \dfrac{\overrightarrow{OB}\,.\,\overrightarrow{OM}}{\lvert\overrightarrow{OB}\rvert\lvert\overrightarrow{OM}\rvert}$ ✗ •5

$\overrightarrow{OB}\,.\,\overrightarrow{OM} = 8$ ✓̸ •6

$\lvert\overrightarrow{OB}\rvert = \sqrt{32}$ ✓̸ •7

$\lvert\overrightarrow{OM}\rvert = 2$ ✓̸✓̸ •8

45° ✓̸ •9

3 marks out of 5

Response 3B

$\cos B\hat{O}D = \dfrac{\overrightarrow{OB}\,.\,\overrightarrow{OD}}{\lvert\overrightarrow{OB}\rvert\lvert\overrightarrow{OD}\rvert}$ ✗ •5

$\overrightarrow{OB}\,.\,\overrightarrow{OD} = 16$ ✓̸ •6

$\lvert\overrightarrow{OB}\rvert = \sqrt{32}$ ✓̸ •7

$\lvert\overrightarrow{OD}\rvert = \sqrt{44}$ ✓̸ •8

$64\cdot8^\circ$ ✓̸ •9

4 marks out of 5

Response 3C

$\cos D\hat{B}M = \dfrac{\overrightarrow{BD}\,.\,\overrightarrow{BM}}{\lvert\overrightarrow{BD}\rvert\lvert\overrightarrow{BM}\rvert}$ ✗ •5

$\overrightarrow{BD}\,.\,\overrightarrow{BM} = 12$ ✓̸ •6

$\lvert\overrightarrow{BD}\rvert = \sqrt{44}$ ✓̸ •7

$\lvert\overrightarrow{BM}\rvert = \sqrt{20}$ ✓̸ •8

$66\cdot1^\circ$ ✓̸ •9

4 marks out of 5

Response 4

$\cos B\hat{D}M = \dfrac{\overrightarrow{BD}\,.\,\overrightarrow{DM}}{\lvert\overrightarrow{BD}\rvert\lvert\overrightarrow{DM}\rvert}$ ✗ •5

$\overrightarrow{BD}\,.\,\overrightarrow{DM} = -32$ ✓̸ •6

$\lvert\overrightarrow{BD}\rvert = \sqrt{44}$ ✓̸ •7

$\lvert\overrightarrow{DM}\rvert = \sqrt{40}$ ✓ •8

$139\cdot7^\circ$ ✓̸ •9

4 marks out of 5

Response 5A

(Scalar Product is 0)

In 1(b) $\overrightarrow{DB} = \begin{pmatrix}2\\-2\\-6\end{pmatrix}$, $\overrightarrow{DM} = \begin{pmatrix}0\\6\\-2\end{pmatrix}$

$\cos B\hat{D}M = \dfrac{\overrightarrow{DB}\,.\,\overrightarrow{DM}}{\lvert\overrightarrow{DB}\rvert\lvert\overrightarrow{DM}\rvert}$ ✓ •5

$\overrightarrow{DB}\,.\,\overrightarrow{DM} = 0$ ✓̸ •6

$\lvert\overrightarrow{DB}\rvert = \sqrt{44}$ ✓ •7

$\lvert\overrightarrow{DM}\rvert = \sqrt{40}$ ✓ •8

90° ✓̸ •9

5 marks out of 5

Response 5B

In 1(b) $\overrightarrow{DB} = \begin{pmatrix}2\\-2\\-6\end{pmatrix}$, $\overrightarrow{DM} = \begin{pmatrix}0\\6\\-2\end{pmatrix}$

$\cos B\hat{D}M = \dfrac{\overrightarrow{DB}\,.\,\overrightarrow{DM}}{\lvert\overrightarrow{DB}\rvert\lvert\overrightarrow{DM}\rvert}$ ✓ •5

$\overrightarrow{DB}\,.\,\overrightarrow{DM} = 0$ ✓̸ •6

so perpendicular ✓̸ •9

^ •7 ^ •8

3 marks out of 5

Response 6 (Cosine rule)

$\cos B\hat{D}M = \dfrac{DB^2 + DM^2 - BM^2}{2 \times DB \times DM}$ ✓ •5

$DB = \sqrt{44}$ ✓ •6

$DM = \sqrt{40}$ ✓ •7

$BM = \sqrt{20}$ ✓ •8

$40\cdot3^\circ$ ✓ •9

5 marks out of 5

2 Functions f, g and h are defined on the set of real numbers by

- $f(x) = x^3 - 1$ • $g(x) = 3x + 1$ • $h(x) = 4x - 5$

(a) Find $g(f(x))$. **2**

(b) Show that $g(f(x)) + x\,h(x) = 3x^3 + 4x^2 - 5x - 2$. **1**

Generic Scheme			Illustrative Scheme	
2 (a)				
•¹	ic	interpret notation	•¹ $g(x^3 - 1)$	**stated, or implied by •²**
•²	ic	complete process	•² $3(x^3 - 1) + 1$	

Notes

1. $3x^3 - 2$ without working gains only 1 mark.
2. $f(g(x))$ loses •¹ but will gain •² for $(3x+1)^3 - 1$.
3. $f(x) \times g(x)$ loses both •¹ and •².

2

Generic Scheme			Illustrative Scheme	
2 (b)				
•³	ic	substitute and complete	•³ $3(x^3 - 1) + 1 + x(4x - 5)$	
			$= 3x^3 + 4x^2 - 5x - 2$	**stated explicitly**
			or	
			$3(x^3 - 1) + 1 + 4x^2 - 5x$	
			$= 3x^3 + 4x^2 - 5x - 2$	**stated explicitly**
			or	
			$3x^3 - 2 + x(4x - 5)$	
			$= 3x^3 + 4x^2 - 5x - 2$	**stated explicitly**
			or	
			$3x^3 - 2 + 4x^2 - 5x$	
			$= 3x^3 + 4x^2 - 5x - 2$	**stated explicitly**

1

Regularly occurring responses

CAVE : Watch out for erroneous working leading to the required cubic.

Response 1 $3x^3 - 2 + \underline{x(4x + 5)} = 3x^3 + 4x^2 - 5x - 2$ ✗ •³

Response 2 $\underline{3x^3 - 4} + x(4x - 5) = 3x^3 + 4x^2 - 5x - 2$ ✗ •³

Response 3 From (a) $(3x + 1)^3 - 1$

In (b) $\underline{3x^3 + 3} - 1 + x(4x - 5) = 3x^3 + 2 + 4x^2 - 5x$ ✗ •³

$= 3x^3 + 4x^2 - 5x - 2$

Response 4A From (a) $g(f(x)) = 3x^3 - 2$

In (b) $xh(x) = 4x^2 - 5x$

^ $3x^3 + 4x^2 - 5x - 2$ ✗ •³

Response 4B From (a) $g(f(x)) = 3x^3 - 2$

In (b) $\underline{3x^2 - 2} + 4x^2 - 5x$ ✗ •³

$= 3x^3 + 4x^2 - 5x - 2$

As the form of the answer was given in the question, this mark is not available.

Note : •³ is not available to candidates who leave their answer as $3x^3 - 2 + 4x^2 - 5x$.

2 (c) (i) Show that $(x-1)$ is a factor of $3x^3+4x^2-5x-2$.

(ii) Factorise $3x^3+4x^2-5x-2$ fully. **5**

(d) Hence solve $g(f(x))+x\,h(x)=0$. **1**

Generic Scheme			**Illustrative Scheme**	
2 (c)			Method 1 : Using synthetic division	
•4	ss	know to use $x=1$	•4 $\begin{array}{c\|cccc} 1 & 3 & 4 & -5 & -2 \\ \hline \end{array}$	
•5	pd	complete evaluation	•5 $\begin{array}{c\|cccc} 1 & 3 & 4 & -5 & -2 \\ & & 3 & 7 & 2 \\ \hline & 3 & 7 & 2 & 0 \end{array}$	If **only** the word 'factor' appears, it must be linked to the 0 in the table. The link could be as little as 'so', '$\therefore$', '$\rightarrow$', '$\Rightarrow$' or 'hence'. The word 'factor' **only**, with no link, does not gain •6.
•6	ic	state conclusion	•6 "remainder is zero so $(x-1)$ is a factor", accept "$(x-1)$ is a factor"	
•7	ic	find quadratic factor	•7 $3x^2+7x+2$	**stated, or implied by •8**
•8	pd	factorise completely	•8 $(x-1)(3x+1)(x+2)$	**stated explicitly**
			Method 2 : Using substitution and inspection	
			•4 know to use $x=1$	
			•5 $3+4-5-2=0$	
			•6 $(x-1)$ is a factor	
			•7 $(x-1)(3x^2+7x+2)$	**stated, or implied by •8**
			•8 $(x-1)(3x+1)(x+2)$	**stated explicitly**

5

Notes

4. •6 is only available as a consequence of the evidence for •4 and •5.
5. Communication at •6 must be consistent with working at •5.
   i.e. candidate's working must arrive legitimately at zero before •6 is awarded.
   If the remainder is not 0 then an appropriate statement would be '$(x-1)$ is not a factor'.
6. Unacceptable statements : $x=1$ is a factor, $(x+1)$ is a factor, $x=1$ is a root, $(x-1)$ is a root etc.
7. •9 cannot be awarded for solving $3x^3+4x^2-5x-2=0$ in (c).

2 (d)				
•9	ic	interpret and solve equation in (d)	•9 $-2,\ -\frac{1}{3}$ and 1	These must appear explicitly here at (d).

1

Notes

8. From (c) $(x-1)(3x+1)(x+2)$ leading to $x=1,\ x=-\frac{1}{3}$ and $x=-2$ then $(1,\ 0)$, $\left(-\frac{1}{3},\ 0\right)$ and $(-2,\ 0)$ gains •9.

   However, $(x-1)(3x+1)(x+2)$ leading to $(1,\ 0)$, $\left(-\frac{1}{3},\ 0\right)$ and $(-2,\ 0)$ **only** does not gain •9.

9. From (c) $(3x+1)(x+2)$ only, leading to $x=-\frac{1}{3},\ x=-2$ does not gain •9 as equation solved is not a cubic.

3	(a)	A sequence is defined by $u_{n+1} = -\frac{1}{2}u_n$ with $u_0 = -16$.	
		Write down the values of u_1 and u_2.	1
	(b)	A second sequence is given by $4,\ 5,\ 7,\ 11,\ \ldots$.	
		It is generated by the recurrence relation $v_{n+1} = pv_n + q$ with $v_1 = 4$.	
		Find the values of p and q.	3

Generic Scheme			Illustrative Scheme	
3 (a)				
$\bullet^1$	pd	find terms of sequence	$\bullet^1$ $u_1 = 8$ and $u_2 = -4$	Accept "8 and -4"
				1

Generic Scheme			Illustrative Scheme	
3 (b)				
$\bullet^2$	ic	interpret sequence	$\bullet^2$ e.g. $4p+q=5$ **and** $5p+q=7$	
$\bullet^3$	ss	solve for one variable	$\bullet^3$ $p=2$ or $q=-3$	
$\bullet^4$	pd	state second variable	$\bullet^4$ $q=-3$ or $p=2$	3

Notes

1. Candidates may use $7p+q=11$ as one of their equations at $\bullet^2$.
2. Treat equations like $p4+q=5$ or $p(4)+q=5$ as bad form.
3. Candidates should not be penalised for using $u_{n+1} = pu_n + q$.

Regularly occurring responses

Response 1A (No working)

$p=2$ and $q=-3$

or $v_{n+1} = 2v_n - 3$

1 mark out of 3

Response 1B (Only one equation)

$4p+q=5$

$p=2$ and $q=-3$

1 mark out of 3

Response 1C (By verification)

$p=2$ and $q=-3$ (ex nihilo)

$v_2 = 8-3=5$

and $v_3 = 10-3=7$

2 marks out of 3

3 (c) Either the sequence in (a) or the sequence in (b) has a limit.

(i) Calculate this limit.

(ii) Why does the other sequence not have a limit? **3**

Generic Scheme			Illustrative Scheme
3 (c)			
•5	ss	know how to find valid limit	•5 $l=-\frac{1}{2}l$ or $l=\frac{0}{1-\left(-\frac{1}{2}\right)}$
•6	pd	calculate a valid limit only	•6 $l=0$
•7	ic	state reason	•7 outside interval $-1<p<1$

3

Notes

4. Just stating that $l=al+b$ or $l=\frac{b}{1-a}$ is not sufficient for •5.

5. Any calculations based on formulae masquerading as a limit rule cannot gain •5 and •6.

6. For candidates who use '$b=0$', •6 is only available to those who simplify $\frac{0}{\dots}$ to 0.

7. Accept $2>1$ or $p>1$ for •7. This may be expressed in words.

8. Candidates who use a without reference to p or 2 cannot gain •7.

Using $l=\frac{b}{1-a}$, $a=-\frac{1}{2}$ and $b=0$, without substituting, and stating $l=0$, gains •5 but not •6.

Regularly occurring responses

Response 2

$l=\frac{0}{\frac{1}{2}}$ ✗ •5

$l=0$ ✗ •6

See note 5

0 marks out of 3

Response 3A

From (b) $p=\frac{3}{4}$ and $q=2$

In (c) $l=\frac{3}{4}l+2$ and $l=-\frac{1}{2}l$ ✓ •5

$l=8$ $\quad l=0$ ✓ •6

Both limits exist as $-1<\frac{3}{4}<1$ and $-1<-\frac{1}{2}<1$ ✓ •7

3 marks out of 3

Response 3B

From (b) $p=\frac{3}{4}$ and $q=2$

In (c) $l=\frac{3}{4}l+2$ and $l=-\frac{1}{2}l$ ✓ •5

$l=8$ $\quad l=0$ ✓ •6

Impossible ✗ •7

2 marks out of 3

Response 4A

$l=\frac{0}{1-\left(-\frac{1}{2}\right)}$ ✓ •5 and $l=\frac{-3}{1-2}$

$l=0$ $\quad l=3$ ✗ •6

First has limit because $-1<-\frac{1}{2}<1$ ✗ •7

1 mark out of 3

Response 4B

$l=\frac{0}{1-\left(-\frac{1}{2}\right)}$ ✓ •5 and $l=\frac{-3}{1-2}$

$l=0$ $\quad l=3$ ✗ •6

Second sequence has no limit as $-1<2<1$ not true ✓ •7

2 marks out of 3

Response 5A

$l=-\frac{1}{2}l$ and $\cancel{l=2l-3}$ ✓ •5

$l=0$ $\quad \cancel{l=3}$ ✓ •6

1st has limit because $-1<0<1$ ✗ •7

2 marks out of 3

Response 5B

$l=-\frac{1}{2}l$ and $\cancel{l=2l-3}$ ✓ •5

$l=0$ $\quad \cancel{l=3}$ ✓ •6

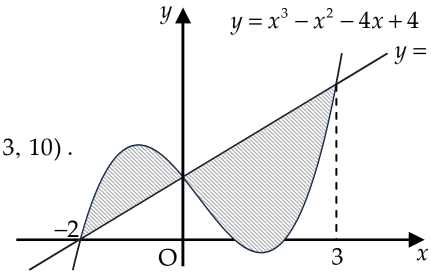
Second sequence has no limit because 2 is not between -1 and 1 ✓ •7

3 marks out of 3

4 The diagram shows the curve with equation $y = x^3 - x^2 - 4x + 4$ and the line with equation $y = 2x + 4$.

The curve and the line intersect at the points $(-2,\ 0)$, $(0,\ 4)$ and $(3,\ 10)$.

Calculate the total shaded area. **10**

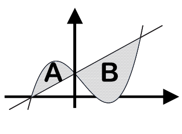


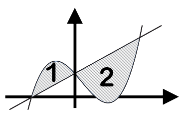
		Generic Scheme	Illustrative Scheme	
4				
$\bullet^1$	ss	know to integrate	$\bullet^1$ $\int \ldots$ **or** attempt integration	
$\bullet^2$	ic	know to deal with areas on each side of $y-$axis	$\bullet^2$ Evidence of attempting to interpret the diagram to left of $y-$axis separately from diagram to the right.	
$\bullet^3$	ic	interpret limits of one area	$\bullet^3$ $\int_{-2}^{0}$	e.g. $\int_{0}^{3}$ with no other
$\bullet^4$	ic	use "upper − lower"	$\bullet^4$ $(x^3 - x^2 - 4x + 4) - (2x + 4)$	$(2x + 4) - (x^3 - x^2 - 4x + 4)$
$\bullet^5$	pd	integrate	$\bullet^5$ $\frac{1}{4}x^4 - \frac{1}{3}x^3 - 3x^2$	$3x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4$
$\bullet^6$	ic	substitute in limits	$\bullet^6$ $-\left(\frac{1}{4}(-2)^4 - \frac{1}{3}(-2)^3 - 3(-2)^2\right)$	$\left(3(3)^2 + \frac{1}{3}(3)^3 - \frac{1}{4}(3)^4\right)$
			Evidence for $\bullet^6$ may be implied by $\bullet^7$, but $\bullet^7$ must be consistent with $\bullet^5$.	
$\bullet^7$	pd	evaluate the area on one side	$\bullet^7$ $\frac{16}{3}$	$\frac{63}{4}$
$\bullet^8$	ss	interpret integrand with limits of the other area	$\bullet^8$ $\int_{0}^{3}(2x + 4) - (x^3 - x^2 - 4x + 4)\,dx$	$\int_{-2}^{0}(x^3 - x^2 - 4x + 4) - (2x + 4)\,dx$
$\bullet^9$	pd	evaluate the area on the other side	$\bullet^9$ $\frac{63}{4}$	$\frac{16}{3}$
$\bullet^{10}$	ic	state total area	$\bullet^{10}$ $21\frac{1}{12}$ or $\frac{253}{12}$ or $21\cdot1$	$21\frac{1}{12}$ or $\frac{253}{12}$ or $21\cdot1$

10

Notes

1. The evidence for $\bullet^2$ may not appear until $\bullet^8$ stage.
2. The evidence for $\bullet^2$ may appear in a diagram e.g.





3. Where a candidate differentiates at $\bullet^5$, then $\bullet^5$, $\bullet^6$, $\bullet^7$ and $\bullet^9$ are not available.
4. Candidates who substitute at $\bullet^6$, without integrating at $\bullet^5$ lose $\bullet^5$, $\bullet^6$ and $\bullet^7$. However $\bullet^8$, $\bullet^9$ and $\bullet^{10}$ are still available.

Regularly occurring responses

General comment to markers

In this question you should scan the entire response before starting to mark. Where errors occur in the integration/evaluation, use $\bullet^3$ to $\bullet^7$ to mark the better solution and $\bullet^8$ and $\bullet^9$ to mark the poorer solution.

A tabular approach to allocating marks is particularly useful in questions like this, where a candidate's response is spread over several pages, or contains working which appears randomly set out. Response 1 indicates the approach to take here.

Response 1

$$\int_{-2}^{3} \text{upper} - \text{lower}$$

$$\int_{-2}^{3} x^3 - x^2 - 6x\,dx$$

$$\frac{1}{4}x^4 - \frac{1}{3}x^3 - 3x^2$$

$$\left(\frac{3^4}{4} - \frac{3^3}{3} - 3.3^2\right) - \left(\frac{-2^4}{4} - \frac{-2^3}{3} - 3.-2^2\right)$$

$$-\frac{125}{12}$$

$\bullet^1$ ✓
$\bullet^2$ ✗
$\bullet^3$ ✗
$\bullet^4$ ✗
$\bullet^5$ ✓
$\bullet^6$ ✓
$\bullet^7$ ✗
$\bullet^8$ ✗
$\bullet^9$ ✗
$\bullet^{10}$ ✗

Response 2

$$\int_{-2}^{0} \ldots = \frac{16}{3}$$

$$\int_{0}^{3} 6x + x^2 - x^3\,dx = \frac{63}{4}$$

$$\int_{1}^{2} x^3 - x^2 - 4x + 4\,dx = -\frac{7}{12}$$

$$\text{Total area} = \frac{16}{3} + \frac{63}{4} + \frac{7}{12} = \frac{260}{12}$$

$\bullet^1$ ✓
$\bullet^2$ ✓
$\bullet^3$ ✓
$\bullet^4$ ✓
$\bullet^5$ ✓
$\bullet^6$ ✓
$\bullet^7$ ✓
$\bullet^8$ ✗
$\bullet^9$ ✗
$\bullet^{10}$ ✗̸

The appearance of this integral is sufficient to lose $\bullet^8$ and $\bullet^9$.

Response 3

$$\int_{1}^{2} x^3 - x^2 - 4x + 4\,dx \quad ✓ \bullet^1$$

$$\frac{1}{4}x^4 - \frac{1}{3}x^3 - 2x^2 + 4x \quad ✓ \bullet^5$$

$$\left(\frac{2^4}{4} - \frac{2^3}{3} - 2.2^2 + 4.2\right) - \left(\frac{1}{4} - \frac{1}{3} - 2 + 4\right) \quad ✓ \bullet^6$$

$$-\frac{7}{12} \quad ✗ \bullet^7$$

Candidates who evaluate an integral and obtain a negative answer must deal with this correctly.
The minimum evidence would be

e.g. $\displaystyle\int_{-2}^{0} \ldots = -\frac{16}{3}$

$$A = \frac{16}{3} \text{ or Area} = \frac{16}{3}$$

N.B. If due to an error the evaluation is negative it must be dealt with correctly. The responses below illustrate what is required under this circumstance. If both integrals lead to negative values only $\bullet^7$ **or** $\bullet^9$ is lost.

Response 4A

$$\int_{0}^{3} \ldots \frac{63}{4}$$

$$\int_{-2}^{0} 6x + x^2 - x^3\,dx \quad ✗ \bullet^8$$

$$= \ldots$$

$$= -\frac{16}{3} \quad ✗ \bullet^9$$

$$\text{Area} = \frac{63}{4} + -\frac{16}{3} \quad ✗ \bullet^{10}$$

$$= \frac{125}{12}$$

Response 4B

$$\int_{0}^{3} \ldots \frac{63}{4}$$

$$\int_{-2}^{0} 6x + x^2 - x^3\,dx \quad ✗ \bullet^8$$

$$= \ldots$$

$$= -\frac{16}{3} \quad ✗ \bullet^9$$

$$\text{Area} = \frac{63}{4} + \frac{16}{3} = \frac{253}{12} \quad ✗̸ \bullet^{10}$$

Response 4C

$$\int_{0}^{3} \ldots \frac{63}{4}$$

$$\int_{-2}^{0} 6x + x^2 - x^3\,dx \quad ✗ \bullet^8$$

$$= \ldots$$

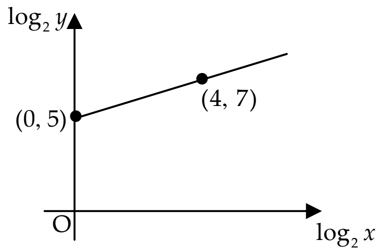
$$= -\frac{16}{3} \quad \text{can't be negative}$$

$$= \frac{16}{3} \quad ✗ \bullet^9$$

$$\text{Area} = \frac{63}{4} + \frac{16}{3} = \frac{253}{12} \quad ✗̸ \bullet^{10}$$

5 Variables x and y are related by the equation $y = kx^n$.

The graph of $\log_2 y$ against $\log_2 x$ is a straight line through the points $(0, 5)$ and $(4, 7)$, as shown in the diagram.



Find the values of k and n. **5**

Generic Scheme			Illustrative Scheme	
5				
Method 1			Method 1	
•1	ss	introduce logarithms to $y = kx^n$	•1 $\log_2 y = \log_2 kx^n$	**stated explicitly**
•2	ic	use laws of logarithms	•2 $\log_2 y = n\log_2 x + \log_2 k$	**stated explicitly**
•3	ic	interpret intercept	•3 $\log_2 k = 5$ **or** $\log_2 y = 5$	Accept without working
•4	ic	solve for k	•4 $k = 32$ or 2^5	
•5	ic	interpret gradient	•5 $n = \frac{1}{2}$	Accept without working
Method 2			Method 2	
•1	ss	state linear equation	•1 $\log_2 y = \frac{1}{2}\log_2 x + 5$	•1 $\log_2 y = \frac{1}{2}\log_2 x + 5$
•2	ic	introduce logarithms	•2 $\ldots + 5\log_2 2$ or $\ldots + \log_2 2^5$	•2 $\log_2 y = \log_2 x^{\frac{1}{2}} + 5$
•3	ic	use laws of logarithms	•3 $\log_2 y = \log_2 x^{\frac{1}{2}} + \ldots$	•3 $\log_2\left(\frac{y}{x^{\frac{1}{2}}}\right) = 5$
•4	ic	use laws of logarithms	•4 $\log_2 y = \log_2 2^5 x^{\frac{1}{2}}$	•4 $\frac{y}{x^{\frac{1}{2}}} = 2^5$
•5	ic	interpret result	•5 $y = 2^5 x^{\frac{1}{2}}$	•5 $y = 2^5 x^{\frac{1}{2}}$
Method 3			Method 3	
•1	ic	interpret point on log. graph	•1 $\log_2 x = 4$ **and** $\log_2 y = 7$	
•2	ic	convert from log. to exp. form	•2 $x = 2^4$ **and** $y = 2^7$	
•3	ic	interpret point and convert	•3 $\begin{cases} \log_2 x = 0 \textbf{ and } \log_2 y = 5 \\ x = 1 \textbf{ and } y = 2^5 \end{cases}$	
•4	ss	know to substitute points	•4 $2^7 = k \times \left(2^4\right)^n$ **and** $2^5 = k$ (from $2^5 = k.1^n$)	
•5	ic	interpret result	•5 $n = \frac{1}{2}$	

5

Notes

1. Omission of base 2 is treated as bad form at the •1 and •2 stage.
2. Gradient $(m) = \frac{1}{2}$ is not sufficient for •5.
3. Throughout this question accept 32 in lieu of 2^5.
4. Markers should not pick and choose within methods. Use the method which gives the candidate the highest mark.

Regularly occurring responses

Response 1A

With no working

$k = 32$ ✓ $\bullet^3$

$n = \frac{1}{2}$ ✓ $\bullet^5$

2 marks out of 5

Response 1B

With no working

$k = \frac{1}{2}$ ✗ $\bullet^3$

$n = 32$ ✗ $\bullet^5$

0 marks out of 5

Response 2 (Method 1)

$\log_2 k = 5$ ✓ $\bullet^3$

$k = 32$ ✓ $\bullet^4$

$n = \frac{1}{2}$ ✓ $\bullet^5$

3 marks out of 5

Response 3 (Variation of Method 2 and Response 1A)

$\log_2 y = \frac{1}{2}\log_2 x + 5$ ✓ $\bullet^1$

$\log_2 y = \log_2 \sqrt{x} + 5$ ✓ $\bullet^2$

$y = \sqrt{x} + 5$

$k = 1,$

$n = \frac{1}{2}$ ✓ $\bullet^5$

3 marks out of 5

Response 4 (Variation of Method 2 and Response 1A)

$y = \frac{1}{2}x + 5$

$\log_2 y = \frac{1}{2}\log_2 x + 5$ ✓ $\bullet^1$

$\log_2 y - \log_2 x^{\frac{1}{2}} = 5$ ✓ $\bullet^2$

$\frac{y}{\sqrt{x}} = 5$ ✗

$y = 5\sqrt{x}$

$k = 5$ ✗

$n = \frac{1}{2}$ ✓ $\bullet^5$

3 marks out of 5

6 (a) The expression $3\sin x - 5\cos x$ can be written in the form $R\sin(x+a)$ where $R>0$ and $0 \le a < 2\pi$.

Calculate the values of R and a. **4**

Generic Scheme			Illustrative Scheme		
6 (a)					
$\bullet^1$	ss	use compound angle formula	$\bullet^1$ $R\sin x\cos a + R\cos x\sin a$		**stated explicitly**
$\bullet^2$	ic	compare coefficients	$\bullet^2$ $R\cos a = 3$ and $R\sin a = -5$		**stated explicitly**
$\bullet^3$	pd	process R	$\bullet^3$ $\sqrt{34}$	(Accept $5\cdot8$)	with or without working
$\bullet^4$	pd	process a	$\bullet^4$ $5\cdot253$	(Accept $5\cdot3$)	must be consistent with $\bullet^2$

4

Notes

1. Treat as bad form the use of k instead of R.
2. Treat $R\sin x\cos a + \cos x\sin a$ as bad form only if the equations at the $\bullet^2$ stage both contain R.
3. $\sqrt{34}\sin x\cos a + \sqrt{34}\cos x\sin a$ or $\sqrt{34}(\sin x\cos a + \cos x\sin a)$ is acceptable for $\bullet^1$ and $\bullet^3$.
4. $\bullet^2$ is not available for $R\cos x = 3$ and $R\sin x = -5$, however, $\bullet^4$ is still available.
5. $\bullet^4$ is only available for a single value of a.
6. Candidates who work in degrees and don't convert to radian measure lose $\bullet^4$. Do not accept $\frac{301\pi}{180}$ or $\frac{5\pi}{3}$.
7. Candidates may use any form of the wave equation for $\bullet^1$, $\bullet^2$ and $\bullet^3$, however, $\bullet^4$ is only available if the value of a is interpreted for the form $R\sin(x+a)$.

Regularly occurring responses

For $\bullet^2$ and $\bullet^4$

Response 1A

$R\cos a = 3 \quad R\sin a = 5$ ✗ $\bullet^2$

$\tan a = \frac{5}{3}$

$a = 1\cdot03$ ✓✗ $\bullet^4$

Response 1B

$R\cos a = 3 \quad R\sin a = 5$ ✗ $\bullet^2$

$\tan a = \frac{3}{5}$

$a = 0\cdot54$ ✗ $\bullet^4$

Response 1C

$R\cos a = 3 \quad R\sin a = -5$ ✓ $\bullet^2$

$\tan a = -\frac{3}{5}$

$a = 5\cdot74$ ✗ $\bullet^4$

Response 2

$R\sin(x-a) = R\sin x\cos a - R\cos x\sin a$ ✓✗ $\bullet^1$

$R\cos a = 3 \quad R\sin a = 5$ ✓✗ $\bullet^2$

$R = \sqrt{34}$ ✓ $\bullet^3$

$a = 1\cdot03$ ✓✗✗ $\bullet^4$

See note 7

3 marks out of 4

Response 3

$k\sin x\cos a + k\cos x\sin a$ ✓ $\bullet^1$

$\cos a = 3 \quad \sin a = -5$ ✗ $\bullet^2$

$R = \sqrt{34}$ ✓✗ $\bullet^3$

$a = 5\cdot3$ ✓✗✗ $\bullet^4$

Not consistent with working at $\bullet^2$

2 marks out of 4

6 (b) Hence find the value of t, where $0 \le t \le 2$, for which

$$\int_0^t (3\cos x + 5\sin x)\,dx = 3$$

7

Generic Scheme			Illustrative Scheme
6 (b)			
•5	pd	integrate given expression	•5 $3\sin x - 5\cos x$
•6	ic	substitute limits	•6 $(3\sin t - 5\cos t) - (3\sin 0 - 5\cos 0)$
•7	pd	process limits	•7 $3\sin t - 5\cos t + 5$
•8	ss	know to use wave equation	•8 $\sqrt{34}\sin(t + 5\cdot 3) + 5$
•9	ic	write in standard format	•9 $\sin(t + 5\cdot 3) = -\frac{2}{\sqrt{34}}$
•10	ss	start to solve equation	•10 $t + 5\cdot 3 = 3\cdot 5$ and $5\cdot 9$
•11	pd	complete and state solution	•11 $t = 0\cdot 6$

•5 to •11 are available to candidates who chose to write this integrand as new wave function.

7

Notes

8. The inclusion of "+ c" at •5 stage should be treated as bad form.
9. For those candidates who use a as $5\cdot 253$ or $5\cdot 25...$, follow through their working for •8 to •11.
10. Candidates who use degree measure in (a) lose •4 and if they continue to do so in (b), only •5, •6, •7 and •8 are available (see also response 6A and 6B below.)

Regularly occurring responses

Response 4 (No integration)

$\int_0^t 3\cos x + 5\sin x\,dx = \sqrt{34}\sin(x + 5\cdot 3)$

lose •5, •6, •7 and •8

then

$\sin(x + 5\cdot 3) = \frac{3}{\sqrt{34}}$ ✗ •9

$x + 5\cdot 3 = 0\cdot 5,\ 2\cdot 6,\ 6\cdot 8$ ✗ •10

$x = 1\cdot 5$ ✗ •11

Needs to be in terms of t

Response 5

$\ldots 3\sin x - 5\cos x$ ✓ •5

$3\sin t - 5\cos t - 0$ ✗ •6 ✗ •7

$\sqrt{34}\sin(t + 5\cdot 3)$ ✗ •8

$\sin(t + 5\cdot 3) = \frac{3}{\sqrt{34}}$ ✗ •9

$t + 5\cdot 3 = 0\cdot 5,\ 2\cdot 6,\ 6\cdot 8$ ✗ •10

$t = 1\cdot 5$ ✗ •11

Response 6A (Misreading question)

$\int \sqrt{34}\sin(x + 5\cdot 3)\,dx$ ✗ •5 ^ •8

$= -\sqrt{34}\cos(t + 5\cdot 3) = 3$ ✗ •6 ✗ •7

$\cos(t + 5\cdot 3) = -\frac{3}{\sqrt{34}}$ ✗ •9

$t + 5\cdot 3 = 2\cdot 1,\ 4\cdot 2,\ 8\cdot 4$ ✗ •10

$t = 3\cdot 1$ i.e. no solution ✗ •11

3 marks out of 7

If a is left in degrees no marks are available.

Response 6B (Misreading question)

$\int \sqrt{34}\sin(x + 5\cdot 3)\,dx$ ✗ •5 ^ •8

$= \left[-\sqrt{34}\cos(x + 5\cdot 3)\right]_0^t$

$= -\sqrt{34}\cos(t + 5\cdot 3) + \sqrt{34}\cos 5\cdot 3$ ✗ •6

$-\sqrt{34}\cos(t + 5\cdot 3) + 3\cdot 2$ ✗ •7

$\cos(t + 5\cdot 3) = \frac{-0\cdot 2}{-\sqrt{34}}$ ✗ •9

$t + 5\cdot 3 = 1\cdot 5,\ 4\cdot 7,\ 7\cdot 8$ ✗ •10

$t = 2\cdot 5$ i.e. no solution ✗ •11

5 marks out of 7

7	Circle C_1 has equation $(x+1)^2 + (y-1)^2 = 121$. A circle C_2 with equation $x^2 + y^2 - 4x + 6y + p = 0$ is drawn inside C_1. The circles have no points of contact. What is the range of values of p?	9

Generic Scheme | **Illustrative Scheme**

7

		Generic Scheme		Illustrative Scheme	
•1	ic	state centre of C_1	•1	$(-1, 1)$	
•2	ic	state radius of C_1	•2	11	Do not accept $\sqrt{121}$
•3	ic	state centre of C_2	•3	$(2, -3)$	
•4	pd	find radius of C_2 in terms of p	•4	$\sqrt{13-p}$	Accept c in lieu of p
•5	ic	interpret upper bound for p	•5	$p < 13$	
•6	ic	find distance between centres (d)	•6	5	**stated explicitly**
•7	ss	identify relevant relationship	•7	$\sqrt{13-p} < 6$ or $r_2 + d < 11$ or $r_2 < 6$	
•8	ic	develop relationship by squaring	•8	$13 - p < 36$	
•9	pd	find lower bound for p	•9	$p > -23$	

9

Notes

1. Treat as bad form the use of c in lieu of p.
2. The evidence for •7 must involve an inequality, but may be in words.
3. Treat $\sqrt{13} - p$ as bad form as long as it is clear that the candidate is using $\sqrt{13-p}$.
4. Candidates who are only working with an equation lose both •7 and •9, however, •8 may still be available.
5. •9 is only available to candidates who solve an inequation involving a negative coefficient of p.

Regularly occurring responses

Response 1A

Marks 1 to 3 gained

^ •5 ^ •6

$\sqrt{-2^2+3^2-p} < 11$

$\sqrt{13-p} < 11$ ✗ •7 ✓ •4

$13 - p < 121$ ✗ •8

$p > -108$ ✗ •9

Response 1B

$C_1 = (-1, 1)$ ✓ •1 $C_2 = (2, -3)$ ✓ •3

$r_1 = 11$ ✓ •2 $r_2 = \sqrt{13+p}$ ✗ •4

$d = 5$ ✓ •6

$\sqrt{13+p} < 11$ ✗ •7

$13 + p < 121$ ✗ •8

$p < 108$ ✗ •9

Response 2

For marks 7 to 9

$\sqrt{13-p} < 6$ ✓ •7

$\sqrt{13} - \sqrt{p} < 6$ ✗ •8

$\underline{169 - p < 36}$

$-p < -133$

$p > 133$ ✗ •9

Penalise the use of $\le$ and/or $\ge$ once only.

Response 3 (see note 4)

$\sqrt{13-p} = 0$

$p = 13$ ✗ •5

$\sqrt{13-p} = 6$ ✗ •7

$13 - p = 36$ ✗ •8

$p > -23$ ✗ •9

Response 4

$\sqrt{13-p} \ge 0$

$p \le 13$ ✗ •5

$\sqrt{13-p} \le 6$ ✗ •7

$p \ge -23$ ✗ •9

Response 5

$0 < \sqrt{13-p} < 6$ ✓•7

$0 < 13 - p < 36$ ✓•8

$-13 < -p < 23$

✓•5

so $p < 13$ and $p > -23$ ✓•9

✓•9

or $-23 < p < 13$ ✓•5

Regularly occurring responses

Response 6

$(x-2)^2+(y+3)^2=13-p$

$13-p<121$ $\bullet^4$ $\bullet^7$

$p>-108$ $\bullet^9$