X100/301

NATIONAL QUALIFICATIONS 2009

THURSDAY, 21 MAY 9.00 AM – 10.30 AM

MATHEMATICS HIGHER
Paper 1
(Non-calculator)

Read carefully

Calculators may NOT be used in this paper.

Section A – Questions 1–20 (40 marks)

Instructions for completion of **Section A** are given on page two.

For this section of the examination you must use an **HB pencil**.

Section B (30 marks)

1 Full credit will be given only where the solution contains appropriate working.

2 Answers obtained by readings from scale drawings will not receive any credit.

SQA ©

Read carefully

1 Check that the answer sheet provided is for **Mathematics Higher (Section A)**.

2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.

3 Check that the answer sheet you have been given has **your name**, **date of birth**, **SCN** (Scottish Candidate Number) and **Centre Name** printed on it.

Do not change any of these details.

4 If any of this information is wrong, tell the Invigilator immediately.

5 If this information is correct, **print** your name and seat number in the boxes provided.

6 The answer to each question is **either** A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space provided (see sample question below).

7 There is **only one correct** answer to each question.

8 Rough working should **not** be done on your answer sheet.

9 At the end of the exam, put the **answer sheet for Section A inside the front cover of your answer book**.

Sample Question

A curve has equation $y = x^3 - 4x$.

What is the gradient at the point where $x = 2$?

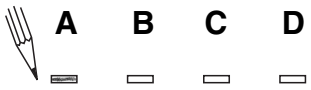
A 8

B 1

C 0

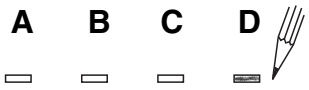
D −4

The correct answer is **A—**8. The answer **A** has been clearly marked in **pencil** with a horizontal line (see below).



Changing an answer

If you decide to change your answer, carefully erase your first answer and, using your pencil, fill in the answer you want. The answer below has been changed to **D**.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned} \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin 2A &= 2\sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

[Turn over

SECTION A

ALL questions should be attempted.

1. A sequence is defined by $u_{n+1} = 3u_n + 4$ with $u_1 = 2$.

What is the value of u_3?

A 34

B 21

C 18

D 13

2. A circle has equation $x^2 + y^2 + 8x + 6y - 75 = 0$.

What is the radius of this circle?

A 5

B 10

C $\sqrt{75}$

D $\sqrt{175}$

3. Triangle PQR has vertices at P(–3, –2), Q(–1, 4) and R(3, 6).

PS is a median. What is the gradient of PS?

A -2

B $-\frac{7}{4}$

C 1

D $\frac{7}{4}$

4. A curve has equation $y = 5x^3 - 12x$.

What is the gradient of the tangent at the point (1, –7)?

A –7

B –5

C 3

D 5

5. Here are two statements about the points S(2, 3) and T(5, –1):

(1) The length of ST = 5 units;

(2) The gradient of ST $= \frac{4}{3}$.

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

6. A sequence is generated by the recurrence relation $u_{n+1} = 0{\cdot}7u_n + 10$.

What is the limit of this sequence as $n \to \infty$?

A $\frac{100}{3}$

B $\frac{100}{7}$

C $\frac{17}{100}$

D $\frac{3}{10}$

7. If the exact value of $\cos x$ is $\frac{1}{\sqrt{5}}$, find the exact value of $\cos 2x$.

A $-\frac{3}{5}$

B $-\frac{2}{\sqrt{5}}$

C $\frac{2}{\sqrt{5}}$

D $\frac{3}{5}$

[Turn over

8. What is the derivative of $\frac{1}{4x^3}$, $x \neq 0$?

A $\frac{1}{12x^2}$

B $-\frac{1}{12x^2}$

C $\frac{4}{x^4}$

D $-\frac{3}{4x^4}$

9. The line with equation $y = 2x$ intersects the circle with equation $x^2 + y^2 = 5$ at the points J and K.

What are the x-coordinates of J and K?

A $x_J = 1, x_K = -1$

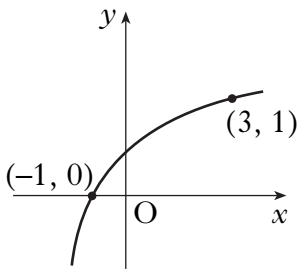
B $x_J = 2, x_K = -2$

C $x_J = 1, x_K = -2$

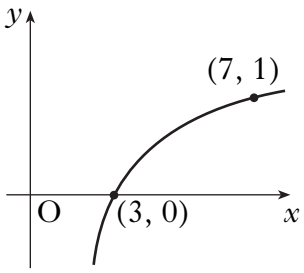
D $x_J = -1, x_K = 2$

10. Which of the following graphs has equation $y = \log_5(x - 2)$?

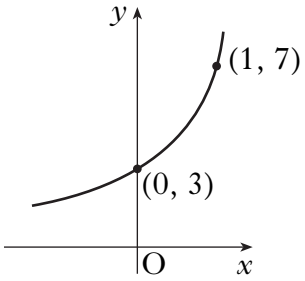
A



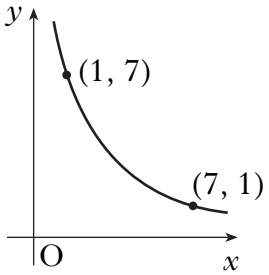
B



C



D



[Turn over

11. How many solutions does the equation

$$(4 \sin x - \sqrt{5})(\sin x + 1) = 0$$

have in the interval $0 \leq x < 2\pi$?

A 4

B 3

C 2

D 1

12. A function f is given by $f(x) = 2x^2 - x - 9$.

Which of the following describes the nature of the roots of $f(x) = 0$?

A No real roots

B Equal roots

C Real distinct roots

D Rational distinct roots

13. k and a are given by

$$k \sin a^\circ = 1$$
$$k \cos a^\circ = \sqrt{3}$$

where $k > 0$ and $0 \leq a < 90$.

What are the values of k and a?

	k	a
A	2	60
B	2	30
C	$\sqrt{10}$	60
D	$\sqrt{10}$	30

14. If $f(x) = 2\sin\left(3x - \frac{\pi}{2}\right) + 5$, what is the range of values of $f(x)$?

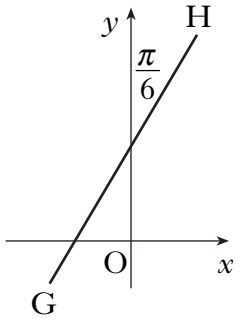
A $-1 \leq f(x) \leq 11$

B $2 \leq f(x) \leq 8$

C $3 \leq f(x) \leq 7$

D $-3 \leq f(x) \leq 7$

15. The line GH makes an angle of $\frac{\pi}{6}$ radians with the y-axis, as shown in the diagram.
What is the gradient of GH?



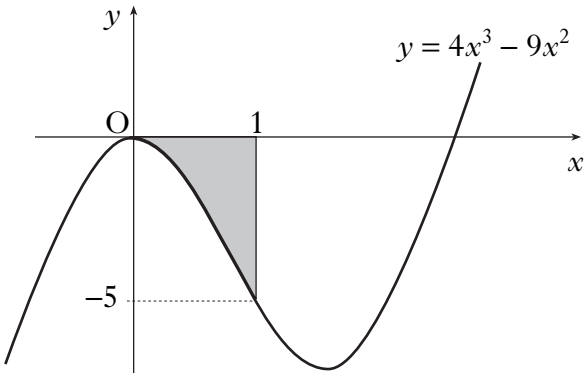
A $\sqrt{3}$

B $\frac{1}{2}$

C $\frac{1}{\sqrt{2}}$

D $\frac{\sqrt{3}}{2}$

16. The graph of $y = 4x^3 - 9x^2$ is shown in the diagram.
Which of the following gives the area of the shaded section?



A $\left[x^4 - 3x^3\right]_{-5}^{0}$

B $-\left[x^4 - 3x^3\right]_{0}^{1}$

C $\left[12x^2 - 18x\right]_{-5}^{0}$

D $-\left[12x^2 - 18x\right]_{0}^{1}$

17. The vector $\boldsymbol{u}$ has components $\begin{pmatrix} -3 \\ 0 \\ 4 \end{pmatrix}$.

Which of the following is a unit vector parallel to $\boldsymbol{u}$?

A $-\frac{3}{5}\boldsymbol{i} + \frac{4}{5}\boldsymbol{k}$

B $-3\boldsymbol{i} + 4\boldsymbol{k}$

C $-\frac{3}{\sqrt{7}}\boldsymbol{i} + \frac{4}{\sqrt{7}}\boldsymbol{k}$

D $-\frac{1}{3}\boldsymbol{i} + \frac{1}{4}\boldsymbol{k}$

18. Given that $f(x) = (4 - 3x^2)^{-\frac{1}{2}}$ on a suitable domain, find $f'(x)$.

A $-3x(4 - 3x^2)^{-\frac{1}{2}}$

B $-\frac{1}{2}(4 - 6x)^{-\frac{3}{2}}$

C $2(4 - 3x^3)^{\frac{1}{2}}$

D $3x(4 - 3x^2)^{-\frac{3}{2}}$

19. For what values of x is $6 + x - x^2 < 0$?

A $x > 3$ only

B $x < -2$ only

C $x < -2$, $x > 3$

D $-3 < x < 2$

20. $A = 2\pi r^2 + 6\pi r$.

What is the rate of change of A with respect to r when $r = 2$?

A 10π

B 12π

C 14π

D 20π

[*END OF SECTION A*]

SECTION B

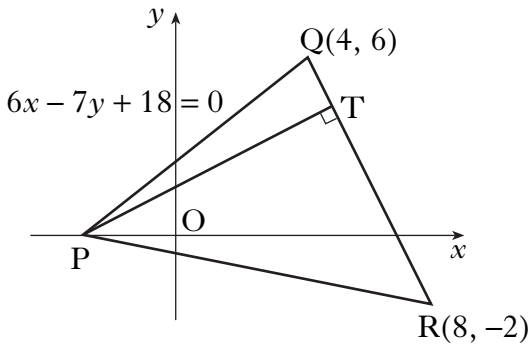
ALL questions should be attempted.

Marks

21. Triangle PQR has vertex P on the x-axis, as shown in the diagram.

Q and R are the points (4, 6) and (8, –2) respectively.

The equation of PQ is $6x - 7y + 18 = 0$.



(*a*) State the coordinates of P. **1**

(*b*) Find the equation of the altitude of the triangle from P. **3**

(*c*) The altitude from P meets the line QR at T. Find the coordinates of T. **4**

22. D, E and F have coordinates (10, –8, –15), (1, –2, –3) and (–2, 0, 1) respectively.

(*a*) (i) Show that D, E and F are collinear.

(ii) Find the ratio in which E divides DF. **4**

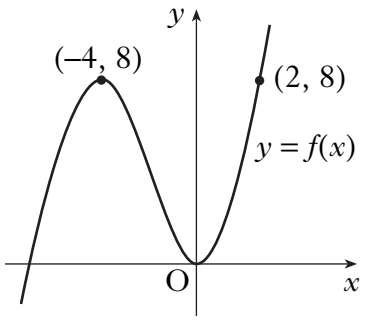
(*b*) G has coordinates (k, 1, 0).

Given that DE is perpendicular to GE, find the value of k. **4**

23. The diagram shows a sketch of the function $y = f(x)$.

(*a*) Copy the diagram and on it sketch the graph of $y = f(2x)$. **2**

(*b*) On a separate diagram sketch the graph of $y = 1 - f(2x)$. **3**



[Turn over for Question 24 on *Page twelve*

Marks

24. (*a*) Using the fact that $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$, find the exact value of $\sin\left(\frac{7\pi}{12}\right)$. **3**

(*b*) Show that $\sin(A + B) + \sin(A - B) = 2\sin A \cos B$. **2**

(*c*) (i) Express $\frac{\pi}{12}$ in terms of $\frac{\pi}{3}$ and $\frac{\pi}{4}$.

(ii) Hence or otherwise find the exact value of $\sin\left(\frac{7\pi}{12}\right) + \sin\left(\frac{\pi}{12}\right)$. **4**

[*END OF SECTION B*]

[*END OF QUESTION PAPER*]

X100/302

NATIONAL QUALIFICATIONS 2009

THURSDAY, 21 MAY
10.50 AM – 12.00 NOON

MATHEMATICS
HIGHER
Paper 2

Read Carefully

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©

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Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

ALL questions should be attempted.

Marks

1. Find the coordinates of the turning points of the curve with equation $y = x^3 - 3x^2 - 9x + 12$ and determine their nature. **8**

2. Functions f and g are given by $f(x) = 3x + 1$ and $g(x) = x^2 - 2$.

(a) (i) Find $p(x)$ where $p(x) = f(g(x))$.

(ii) Find $q(x)$ where $q(x) = g(f(x))$. **3**

(b) Solve $p'(x) = q'(x)$. **3**

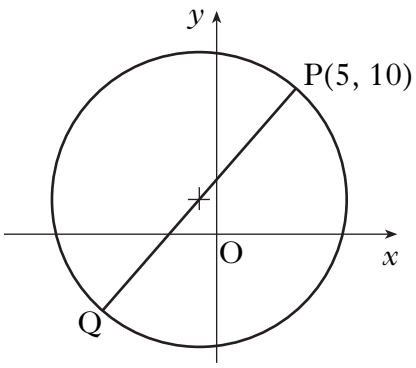
3. (a) (i) Show that $x = 1$ is a root of $x^3 + 8x^2 + 11x - 20 = 0$.

(ii) Hence factorise $x^3 + 8x^2 + 11x - 20$ fully. **4**

(b) Solve $\log_2(x + 3) + \log_2(x^2 + 5x - 4) = 3$. **5**

4. (a) Show that the point P(5, 10) lies on circle C_1 with equation $(x + 1)^2 + (y - 2)^2 = 100$. **1**

(b) PQ is a diameter of this circle as shown in the diagram. Find the equation of the tangent at Q. **5**



(c) Two circles, C_2 and C_3, touch circle C_1 at Q.

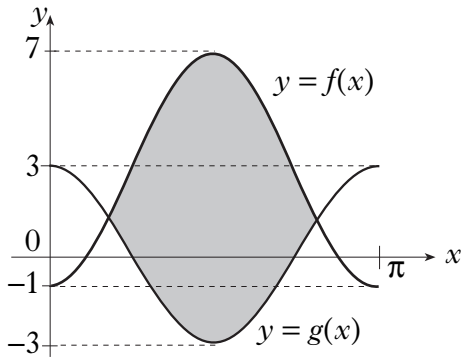
The radius of each of these circles is twice the radius of circle C_1.

Find the equations of circles C_2 and C_3. **4**

[Turn over

Marks

5. The graphs of $y = f(x)$ and $y = g(x)$ are shown in the diagram.



$f(x) = -4\cos(2x) + 3$ and $g(x)$ is of the form $g(x) = m\cos(nx)$.

(*a*) Write down the values of m and n. **1**

(*b*) Find, correct to one decimal place, the coordinates of the points of intersection of the two graphs in the interval $0 \leq x \leq \pi$. **5**

(*c*) Calculate the shaded area. **6**

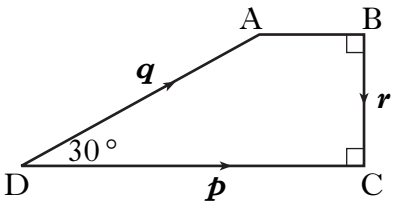
6. The size of the human population, N, can be modelled using the equation $N = N_0e^{rt}$ where N_0 is the population in 2006, t is the time in years since 2006, and r is the annual rate of increase in the population.

(*a*) In 2006 the population of the United Kingdom was approximately 61 million, with an annual rate of increase of 1·6%. Assuming this growth rate remains constant, what would be the population in 2020? **2**

(*b*) In 2006 the population of Scotland was approximately 5·1 million, with an annual rate of increase of 0·43%.

Assuming this growth rate remains constant, how long would it take for Scotland's population to double in size? **3**

7. Vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are represented on the diagram shown where angle ADC = 30°.



It is also given that $|\boldsymbol{p}| = 4$ and $|\boldsymbol{q}| = 3$.

(*a*) Evaluate $\boldsymbol{p}.(\boldsymbol{q} + \boldsymbol{r})$ and $\boldsymbol{r}.(\boldsymbol{p} - \boldsymbol{q})$. **6**

(*b*) Find $|\boldsymbol{q} + \boldsymbol{r}|$ and $|\boldsymbol{p} - \boldsymbol{q}|$. **4**

[*END OF QUESTION PAPER*]