H

National
Qualifications
2026

2026 Mathematics

Higher – Paper 1 (Non-calculator)

Question Paper Finalised Marking Instructions

Qualifications Scotland
Teisteanasan Alba

General marking principles for Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example 6 × 6 = 12, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$x = 1$$

This is no longer a solution of a quadratic equation, so the mark is not awarded.

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$(x - 3)(x - 1) = 0$$
$$x = 1 \text{ or } 3$$

(i) Horizontal/vertical marking
If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x=2$	$x=-4$
$\bullet^6$	$y=5$	$y=-7$

Horizontal: $\bullet^5$ $x=2$ and $x=-4$
$\bullet^6$ $y=5$ and $y=-7$

Vertical: $\bullet^5$ $x=2$ and $y=5$
$\bullet^6$ $x=-4$ and $y=-7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0\cdot3}$ must be simplified to 50 $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*****

*The square root of perfect squares up to and including 144 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $\left(x^3+2x^2+3x+2\right)\left(2x+1\right)$ written as
  $\left(x^3+2x^2+3x+2\right)\times 2x+1$
  $=2x^4+5x^3+8x^2+7x+2$
  gains full credit

- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Marking instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.			**Method 1** •1 identify common factor •2 complete the square •3 process for r and write in required form	**Method 1** •1 $2(x^2+10x\ldots$ stated or implied by •2 •2 $2(x+5)^2\ldots$ •3 $2(x+5)^2-47$	3
			Method 2 •1 expand completed square form •2 equate coefficients •3 process for q and r and write in required form	**Method 2** •1 $px^2+2pqx+pq^2+r$ stated or implied by •2 •2 $p=2$, $2pq=20$, $pq^2+r=3$ •3 $2(x+5)^2-47$	

Notes:

1. $2(x+5)^2-47$ with no working gains •1 and •2 only. However, see Candidate E.
2. In Method 2, do not penalise candidates who do not work with p, q and r.

Commonly Observed Responses:

Candidate A $2(x^2+10)+3$ $2\left((x+5)^2-25\right)+3$ •1 ✓ •2 ✓ $2(x+5)^2-47$ •3 ✓ See exception to general marking principle (h)	**Candidate B - not using required form** $px^2+2pqx+pq^2+r$ •1 ✓ $p=2$, $2pq=20$, $pq^2+r=3$ •2 ✓ $q=5$, $r=-47$ •3 ^ •3 is lost as answer is not in completed square form
Candidate C $2(x^2+20x)+3$ •1 ✗ $2\left((x+10)^2-100\right)+3$ •2 ✓1 $2(x+10)^2-197$ •3 ✓1	**Candidate D** $2\left((x+10)^2-100\right)+3$ •1 ✗ •2 ✗ $2(x+10)^2-197$ •3 ✓1
Candidate E $2(x+5)^2-47$ •1 ✓ •2 ✓ Check: $=2(x^2+10x+25)-47$ $=2x^2+20x+50-47$ $=2x^2+20x+3$ •3 ✓	**Candidate F** $2(x+5)^2=2x^2+20x+50$ •1 ✓ •2 ✓ $\therefore 2(x+5)^2-47=2x^2+20x+3$ •3 ✓

Question			Generic scheme	Illustrative scheme	Max mark
2.			•¹ express second term in integrable form •² integrate first term •³ integrate second term •⁴ complete integration	•¹ $\ldots + 7x^{-2}$ •² $\dfrac{15x^{\frac{5}{3}}}{\frac{5}{3}}$ •³ $+\dfrac{7x^{-1}}{-1}$ •⁴ $9x^{\frac{5}{3}} - 7x^{-1} + c$	4

Notes:

1. •¹ may be implied by the appearance of $-\dfrac{7x^{-1}}{1}$ or $-\dfrac{7}{x}$.
2. Do not penalise the appearance of '$+c$', missing integral signs or missing 'dx' at •¹.
3. •³ is only available for integrating a term with a negative power.
4. Do not penalise the appearance of an integral sign and/or dx throughout.
5. Do not penalise the omission of '$+c$' at •² or •³.
6. All coefficients must be simplified at •⁴ stage for •⁴ to be awarded.
7. •⁴ is only available for attempting to integrate both terms.
8. Accept $9x^{\frac{5}{3}} + \dfrac{7}{-x} + c$ for •⁴. However, do not accept $9x^{\frac{5}{3}} + \dfrac{7}{-1x} + c$ or $9x^{\frac{5}{3}} + -\dfrac{7}{x} + c$ for •⁴.

Question		Generic scheme	Illustrative scheme	Max mark
2.	(continued)			

Commonly Observed Responses:

Candidate A - Integrating over two lines $\frac{15x^{\frac{5}{3}}}{\frac{5}{3}}+7x^{-2}$ •1 ✓ $\frac{15x^{\frac{5}{3}}}{\frac{5}{3}}+\frac{7x^{-1}}{-1}$ •2 ✗ •3 ✓ $9x^{\frac{5}{3}}-7x^{-1}+c$ •4 ✓1	**Candidate B - error in integration** $\int\left(15x^{\frac{2}{3}}+7x^{-2}\right)dx$ •1 ✓ $\frac{15x^{\frac{5}{3}}}{\frac{5}{3}}+\frac{7x^{-3}}{-3}$ •2 ✓ •3 ✗ $9x^{\frac{5}{3}}-\frac{7}{3}x^{-3}+c$ •4 ✓1
Candidate C - +c appears on a fully simplified line of working $9x^{\frac{5}{3}}-7x^{-1}$ •2 ✓ •3 ✓ $9\sqrt[3]{x^5}-\frac{7}{x}+c$ •4 ✓	**Candidate D - simplification never fully correct** $\frac{15x^{\frac{5}{3}}}{\frac{5}{3}}+\frac{7x^{-1}}{-1}$ •2 ✓ •3 ✓ $9x^{\frac{5}{3}}+\boxed{\frac{7x^{-1}}{-1}}+c$ $\boxed{9\sqrt[5]{x^3}}-\frac{7}{x}+c$ •4 ✗
Candidate E - +c not present on a fully simplified line of working $\frac{15x^{\frac{5}{3}}}{\frac{5}{3}}+\frac{7x^{-1}}{-1}+c$ •2 ✓ •3 ✓ $9x^{\frac{5}{3}}-7x^{-1}$ •4 ^	

Question			Generic scheme	Illustrative scheme	Max mark
3.			•¹ state coordinates of centre	•¹ $(-1,2)$	4
			•² find gradient of radius	•² $\frac{4}{3}$ or $\frac{-4}{-3}$	
			•³ state perpendicular gradient	•³ $-\frac{3}{4}$	
			•⁴ determine equation of tangent	•⁴ $4y=-3x+30$	

Notes:

1. •⁴ is only available as a consequence of trying to find and use a perpendicular gradient.
2. At •⁴, accept $3x+4y=30$ or any other rearrangement of the equation where the constant terms have been simplified.

Commonly Observed Responses:

Candidate A - unsimplified constant terms

$(-1,2)$	•¹ ✓
$m=\frac{-4}{-3}$	•² ✓
$m_{\perp}=-\frac{3}{4}$	•³ ✓
$y=-\frac{3}{4}x+\frac{30}{4}$	•⁴ ^

However…

$4y=-3x+30$	•⁴ ✓
$y=-\frac{3}{4}x+\frac{30}{4}$	

Candidate B - $m=m_{\perp}$

$(-1,2)$	•¹ ✓
$m=\frac{4}{3}=-\frac{3}{4}$	•² ✓ •³ ✗
$4y=-3x+30$	•⁴ $\checkmark_1$ - BOD

However…

$m=\frac{4}{3}$	•² ✓
$=-\frac{3}{4}$	•³ ✓ - BOD
$4y=-3x+30$	•⁴ ✓

Question			Generic scheme	Illustrative scheme	Max mark
4.	(a)		•1 find $\sin q$	•1 $\frac{12}{13}$	**1**

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)	(i)	•2 find two remaining trig ratios	•2 two from $\sin p = \frac{4}{5}$, $\cos q = \frac{5}{13}$, and $\cos p = \frac{3}{5}$ stated or implied by •3 and •4	**4**
			•3 select appropriate formula and express in terms of p and q	•3 $\sin p \cos q + \cos p \sin q$	
			•4 substitute into addition formula	•4 $\frac{4}{5} \times \frac{5}{13} + \frac{3}{5} \times \frac{12}{13}$	
			•5 find $\sin(p+q)$	•5 $\frac{56}{65}$	
		(ii)	•6 substitute into appropriate formula	•6 $\frac{3}{5} \times \frac{5}{13} - \frac{4}{5} \times \frac{12}{13}$	**2**
			•7 find $\cos(p+q)$	•7 $-\frac{33}{65}$	

Notes:

1. Accept two from $p = \cos^{-1}\left(\frac{3}{5}\right)$, $p = \sin^{-1}\left(\frac{4}{5}\right)$ or $q = \cos^{-1}\left(\frac{5}{13}\right)$ for •2.
2. Where candidates substitute an incorrect value for $\sin p$, $\cos p$ or $\cos q$, •4 and •6 may be awarded if this value has previously been stated or it can be implied by a diagram or Pythagoras calculation. See Candidate A.
3. Where candidates explicitly state a value for $\sin p$, $\cos p$ or $\cos q$, subsequent working must follow from that value for •4 to be awarded.
4. Award •2 and •3 for candidates who write $\sin\left(\frac{4}{5}\right) \times \cos\left(\frac{5}{13}\right) + \cos\left(\frac{3}{5}\right) \times \sin\left(\frac{12}{13}\right)$, •4 and •5 are unavailable.
5. For any attempt to use $\sin(p+q) = \sin p \pm \sin q$, •4 and •5 are unavailable.
6. Do not penalise trigonometric ratios which are less than −1 or greater than 1 throughout this question.
7. •5 is only available for an answer expressed as a single fraction.

Question			Generic scheme	Illustrative scheme	Max mark
4.	(b)	(ii)	**(continued)**		

Commonly Observed Responses:

Candidate A - incorrect use of Pythagoras

Missing sides of $\sqrt{7}$ and $\sqrt{162}$. $\bullet^1$ is unavailable.

Follow through marks are available leading to:

$$\sin(p+q)=\frac{4\sqrt{7}+3\sqrt{162}}{13\sqrt{7}},$$

$$\cos(p+q)=\frac{3\sqrt{7}-4\sqrt{162}}{13\sqrt{7}}, \text{ and}$$

$$\tan(p+q)=\frac{4\sqrt{7}+3\sqrt{162}}{3\sqrt{7}-4\sqrt{162}}$$

Candidate B - incorrect use of Pythagoras

Missing sides of 5 and $\sqrt{194}$. $\bullet^1$ is unavailable.

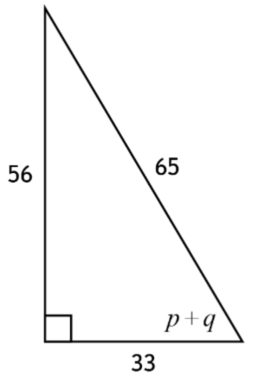
Follow through marks are available leading to:

$$\sin(p+q)=\frac{20+3\sqrt{194}}{65},$$

$$\cos(p+q)=\frac{15-4\sqrt{194}}{65}, \text{ and}$$

$$\tan(p+q)=\frac{20+3\sqrt{194}}{15-4\sqrt{194}}$$

Candidate C - not considering $p+q$ is obtuse



$\bullet^6$ ✗

$$\cos(p+q)=\frac{33}{65}$$

$\bullet^7$ ✓$_1$

Question			Generic scheme	Illustrative scheme	Max mark
4.	(c)		$\bullet^8$ find $\tan(p+q)$	$\bullet^8$ $-\frac{56}{33}$	1

Notes:

8. Accept $-\frac{3640}{2145}$, $-\frac{728}{429}$ or $-\frac{280}{165}$ for $\bullet^8$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
5.	(a)	(i)	$\bullet^1$ interpret notation $\bullet^2$ state expression for $f\left(g\left(x\right)\right)$	$\bullet^1$ $f\left(x+3\right)$ **or** $2\left(g\left(x\right)\right)^2+5$ stated or implied by $\bullet^2$ $\bullet^2$ $2\left(x+3\right)^2+5$	2
		(ii)	$\bullet^3$ state expression for $g\left(f\left(x\right)\right)$	$\bullet^3$ $\left(2x^2+5\right)+3$	1

Notes:

1. Accept $2g\left(x\right)^2+5$ for $\bullet^1$.

Commonly Observed Responses:

Candidate A - two attempts	Candidate B - $f\left(g\left(x\right)\right)$ and $g\left(f\left(x\right)\right)$ interchanged
$f\left(g\left(x\right)\right)=2x^2+5$ $=2\left(x+3\right)^2+5$ $\quad\bullet^1$ ✘ $\bullet^2$ ✘	$f\left(g\left(x\right)\right)=\left(2x^2+5\right)+3$ $\quad\bullet^1$ ✘ $\bullet^2$ $\checkmark_1$ $g\left(f\left(x\right)\right)=2\left(x+3\right)^2+5$ $\quad\bullet^3$ $\checkmark_1$

	(b)		$\bullet^4$ state range	$\bullet^4$ $g\left(f\left(x\right)\right)\geq 8$	1

Notes:

2. Accept $y\geq 8$ or $\left[8,\infty\right)$ for $\bullet^4$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
6.			•¹ state an appropriate pathway •² express pathway in terms of $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$	•¹ eg $\overrightarrow{MH}+\overrightarrow{HE}+\overrightarrow{EN}$ stated **or** implied by •² •² $-\frac{1}{2}\mathbf{u}-\mathbf{v}+\frac{1}{3}\mathbf{w}$	2

Notes:

1. Do not penalise inconsistent vector notation (for example lack of arrows or brackets).
2. Where an invalid pathway has been stated, award 0/2.
3. •¹ may be awarded for the appearance of $\frac{1}{2}\mathbf{u}-\mathbf{u}-\mathbf{v}+\mathbf{w}-\frac{2}{3}\mathbf{w}$. •² is not available for an unsimplified expression.
4. Where no pathway has been stated, see Candidates A and B.

Commonly Observed Responses:

Candidate A - no pathway stated and incorrect expression in $\mathbf{u}$, $\mathbf{v}$ AND $\mathbf{w}$	Candidate B - no pathway stated and incorrect expression in $\mathbf{v}$, $\mathbf{w}$ AND $\mathbf{u}$
$-\frac{1}{2}\mathbf{u}-\mathbf{v}\ldots$ Award 1/2	$\ldots-\mathbf{v}+\frac{1}{3}\mathbf{w}$ Award 1/2

Question			Generic scheme	Illustrative scheme	Max mark
7.			•1 write second term in differentiable form	•1 $\ldots+4x^{\frac{1}{2}}$ stated or implied by •3	4
			•2 differentiate first term	•2 2	
			•3 differentiate second term	•3 $\ldots+\frac{1}{2}\times4x^{-\frac{1}{2}}$	
			•4 evaluate gradient	•4 $\frac{8}{3}$ or $2\frac{2}{3}$	

Notes:

1. •3 is only available for differentiating a term with a fractional index.
2. •4 is only available as a consequence of substituting into an expression involving a fractional power of x.
3. Where candidates make no attempt to differentiate, or integrate one or other of the terms, •4 is unavailable. See Candidates A, B and C.
4. •4 is not available where a candidate subsequently finds a perpendicular gradient.
5. Accept $y=\frac{8}{3}$ only where a substitution has been shown, no other values for y have been stated, or $\frac{8}{3}$ has been used as the gradient in the equation of the tangent.
6. Where candidates equate $\frac{dy}{dx}$ to 0, •4 is unavailable. See Candidate D.

Commonly Observed Responses:

Candidate A - limited evidence of differentiation of the second term		Candidate B - second term integrated	
$y=2x+4x^{\frac{1}{2}}$	•1 ✓	$y=2x+4x^{\frac{1}{2}}$	•1 ✓
$\frac{dy}{dx}=2+2x^{\frac{3}{2}}$	•2 ✓ •3 ✗	$\frac{dy}{dx}=2+\frac{8}{3}x^{\frac{3}{2}}$	•2 ✓ •3 ✗ •4 ✗
$2+2(9)^{\frac{3}{2}}$		$2+\frac{8}{3}(9)^{\frac{3}{2}}$	
$=56$	•4 ✓1	$=74$	

Candidate C - inclusion of $+c$		Candidate D - equating to 0	
$y=2x+4x^{\frac{1}{2}}$	•1 ✓	$y=2x+4x^{\frac{1}{2}}$	•1 ✓
$\frac{dy}{dx}=2+2x^{-\frac{1}{2}}+c$	•2 ✓ •3 ✗ •4 ✗	$\frac{dy}{dx}=2+2x^{-\frac{1}{2}}=0$	•2 ✓ •3 ✓ •4 ✗
$m=\frac{8}{3}$		$m=\frac{8}{3}$	

Question			Generic scheme	Illustrative scheme	Max mark
8.			**Method 1** •¹ apply $\log_2 x + \log_2 y = \log_2 xy$ •² write in exponential form •³ express in standard quadratic form •⁴ solve quadratic and state solution of log equation	**Method 1** •¹ $\log_2 \left(x(x+2)\right) = \ldots$ •² $x(x+2) = 2^3$ •³ $x^2 + 2x - 8 = 0$ •⁴ $-4, 2$ **and** $x > 0 \Rightarrow x = 2$	4
			Method 2 •¹ apply $\log_2 x + \log_2 y = \log_2 xy$ •² apply $m\log_2 x = \log_2 x^m$ •³ express in standard quadratic form •⁴ solve quadratic and state solution of log equation	**Method 2** •¹ $\log_2 \left(x(x+2)\right) = \ldots$ •² $\ldots = \log_2 2^3$ •³ $x^2 + 2x - 8 = 0$ •⁴ $-4, 2$ **and** $x > 0 \Rightarrow x = 2$	

Notes:

1. Accept $\log_2 x(x+2) = \ldots$ or $\log_2 x^2 + 2x = \ldots$ for •¹.
2. •¹ and •² may be awarded for $x(x+2) = 2^3$. However, where 'log' has been scored out, see Candidates C, D and E.
3. Each line of working must be equivalent to the line above within a valid strategy.
4. •² is not available for $x(x+2) = 3^2$. However, candidates may still gain •³ and •⁴. See Candidate B.
5. •³ and •⁴ are only available if the quadratic reached at •³ is obtained by applying the rules in •¹ and •². See Candidates A and B.
6. •⁴ is only available for solving a polynomial of degree two or higher.
7. At •⁴, accept any indication that −4 has been discarded. For example, scoring out $x = -4$ or underlining $x = 2$.
8. Do not penalise the omission of $x > 0$ at •⁴.

Question		Generic scheme	Illustrative scheme	Max mark
8.	(continued)			

Commonly Observed Responses:

Candidate A - $2^3 = 6$		Candidate B	
$\log_2 x(x+2)=3$	$\bullet^1$ ✓	$\log_2 x(x+2)=3$	$\bullet^1$ ✓
$x(x+2)=6$	$\bullet^2$ ✗	$x(x+2)=9$	$\bullet^2$ ✗
$x^2+2x-6=0$	$\bullet^3$ $\checkmark_1$	$x^2+2x-9=0$	$\bullet^3$ $\checkmark_1$
$x=-1\pm\sqrt{7}$		$x=-1\pm\sqrt{10}$	
$x>0\Rightarrow x=\sqrt{7}-1$	$\bullet^4$ $\checkmark_1$	$x>0\Rightarrow x=\sqrt{10}-1$	$\bullet^4$ $\checkmark_1$

Candidate C - valid with and without scoring out		Candidate D - invalid with and without scoring out	
$\cancel{\log}_2 x(x+2)=\cancel{\log}_2 8$	$\bullet^1$ ✓	$\cancel{\log}_2 x(x+2)=\cancel{\log}_2 3$	$\bullet^1$ ✓ - BOD
$x(x+2)=8$	$\bullet^2$ ✓	$x(x+2)=8$	$\bullet^2$ ✗
$x^2+2x-8=0$	$\bullet^3$ ✓	$x^2+2x-8=0$	$\bullet^3$ ✗
$x=\cancel{-4}, 2$	$\bullet^4$ ✓	$x=\cancel{-4}, 2$	$\bullet^4$ ✗
$\bullet^3$ and $\bullet^4$ are available		$\bullet^3$ and $\bullet^4$ are **not** available	

Candidate E - invalid with and without scoring out		Candidate F - invalid processing of logs	
$\cancel{\log}_2 x(x+2)=\cancel{\log}_2 3$	$\bullet^1$ ✓ - BOD	$\log_2 x(x+2)=2^3$	$\bullet^1$ ✓ $\bullet^2$ ✗
$x(x+2)=3$	$\bullet^2$ ✗	$x(x+2)=8$	
$x^2+2x-3=0$	$\bullet^3$ ✗	$x^2+2x-8=0$	$\bullet^3$ ✗
$x=\cancel{-3}, 1$	$\bullet^4$ ✗	$x=\cancel{-4}, 2$	$\bullet^4$ ✗
$\bullet^3$ and $\bullet^4$ are **not** available		$\bullet^3$ and $\bullet^4$ are **not** available	

Question			Generic scheme	Illustrative scheme	Max mark
9.	(a)		•[1] find an appropriate vector eg $\overrightarrow{DE}$ •[2] find a second vector eg $\overrightarrow{EF}$ and compare •[3] appropriate conclusion	•[1] eg $\overrightarrow{DE} = \begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix}$ •[2] eg $\overrightarrow{EF} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \therefore \overrightarrow{DE} = 2\overrightarrow{EF}$ •[3] $\Rightarrow$ DE is parallel to EF (common direction) and E is a common point $\Rightarrow$ D,E and F are collinear	3

Notes:

1. Do not penalise inconsistent vector notation (for example lack of arrows or brackets).
2. The comparison at •[2] may be expressed as a ratio, for example DE : EF = 2 : 1.
3. If no comparison of vectors or the trivial comparison '$\overrightarrow{DE} = \overrightarrow{EF}$' is made at •[2], then •[3] is not available.
4. •[3] can only be awarded if a candidate has stated 'parallel', 'common point' and 'collinear'.
5. Candidates who state or imply that 'points are parallel' or 'vectors are collinear' or state 'parallel and share a common point $\Rightarrow$collinear' do not gain •[3]. There must be reference to the points.
6. Do not accept 'd, e and f are collinear' at •[3].

Commonly Observed Responses:

Candidate A - missing labels

$\begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix}$ •[1] ^

$\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \therefore \overrightarrow{DE} = 2\overrightarrow{EF}$ •[2] ✓[1]

Missing labels treated as a repeated error

$\Rightarrow$ DE is parallel to EF (common direction)
and E is a common point
$\Rightarrow$ D,E and F are collinear •[3] ✓[1]

Candidate B

$\overrightarrow{EF} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ •[1] ✓

$\overrightarrow{DE} = \begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix} = 2\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ •[2] ✓

$\therefore 2\overrightarrow{DE} = \overrightarrow{EF}$

ignore working subsequent to correct statement on previous line

$\Rightarrow$ DE is parallel to EF (common direction)
and E is a common point
$\Rightarrow$ D,E and F are collinear •[3] ✓

Question			Generic scheme	Illustrative scheme	Max mark
9.	(b)		**Method 1**	**Method 1**	**2**
			•⁴ interpret ratio	•⁴ $\begin{pmatrix}3\\-6\\9\end{pmatrix}$ **and** $\begin{pmatrix}4\\-8\\12\end{pmatrix}$ **or** $\begin{pmatrix}7\\-14\\21\end{pmatrix}$	
			•⁵ find coordinates of G	•⁵ $(6,-8,22)$	
			Method 2	**Method 2**	
			•⁴ interpret ratio	•⁴ eg $\overrightarrow{FG}=\frac{4}{3}\overrightarrow{DF}$ **or** $\overrightarrow{DG}=\frac{7}{3}\overrightarrow{DF}$	
			•⁵ find coordinates of G	•⁵ $(6,-8,22)$	
			Method 3	**Method 3**	
			•⁴ use section formula	•⁴ $\frac{1}{3}(-4\mathbf{d}+7\mathbf{f})$ **or** $\mathbf{f}=\frac{3}{7}\mathbf{g}+\frac{4}{7}\mathbf{d}$	
			•⁵ find coordinates of G	•⁵ $(6,-8,22)$	

Notes:

7. Do not penalise inconsistent vector notation (for example lack of arrows or brackets).
8. For $(6,-8,22)$ without working award 2/2.
9. For $\begin{pmatrix}6\\-8\\22\end{pmatrix}$ without working award 1/2.
10. For $\left(\frac{17}{4},-\frac{9}{2},\frac{67}{4}\right)$ (ratio of 4:3 with working) award 1/2. See Candidate C.
11. No marks are available for finding an internal division point, for example $\left(\frac{2}{7},\frac{24}{7},\frac{34}{7}\right)$ or using other ratios.

Question		Generic scheme	Illustrative scheme	Max mark
9.	(b)	(continued)		

Commonly Observed Responses:

<table>
<tr>
<td>

Candidate C - ratio of 4:3

$\overrightarrow{FG}=\frac{3}{4}\overrightarrow{DF}$ $\bullet^4$ ✗

$G=\left(\frac{17}{4},-\frac{9}{2},\frac{67}{4}\right)$ $\bullet^5$ ✓1

</td>
<td>

Candidate D

$\frac{\overrightarrow{FG}}{\overrightarrow{DF}}=\frac{4}{3}$ $\bullet^4$ ✓

$3\overrightarrow{FG}=4\overrightarrow{DF}$

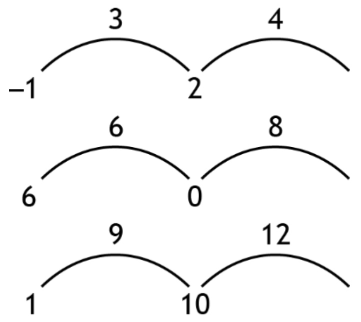
$3(\mathbf{g}-\mathbf{f})=4(\mathbf{f}-\mathbf{d})$

$3\mathbf{g}=7\mathbf{f}-4\mathbf{d}$

G = (6,−8,22) $\bullet^5$ ✓

</td>
</tr>
<tr>
<td>

Candidate E - stepping out using absolute values



$\bullet^4$ ✓

G = (6,−8,22) $\bullet^5$ ✓

</td>
<td>

Candidate F - Component form of section formula

$\frac{4(-1)+3x}{7}=2$

$\frac{4(6)+3y}{7}=0$

$\frac{4(1)+3z}{7}=10$ $\bullet^4$ ✓

G = (6,−8,22) $\bullet^5$ ✓

</td>
</tr>
</table>

Question			Generic scheme	Illustrative scheme	Max mark
10.			$\bullet^1$ start to integrate	$\bullet^1$ $-6\cos\left(3x-\frac{\pi}{2}\right)$	4
			$\bullet^2$ complete integration	$\bullet^2$ $\ldots\times\frac{1}{3}$	
			$\bullet^3$ substitute limits	$\bullet^3$ $\left(-2\cos\left(3\times\frac{\pi}{3}-\frac{\pi}{2}\right)\right)-\left(-2\cos\left(3\times\frac{\pi}{6}-\frac{\pi}{2}\right)\right)$	
			$\bullet^4$ evaluate integral	$\bullet^4$ 2	

Notes:

1. Award $\bullet^1$ for the appearance of $-6\cos\left(3x-\frac{\pi}{2}\right)$ regardless of any **constant** multiplier.
2. For candidates who differentiate throughout, make no attempt to integrate, or use another invalid approach – for example $\cos\left(3x-\frac{\pi}{2}\right)^2$ or $6\int \sin 3x-\sin\frac{\pi}{2}\,dx$ – award 0/4.
3. For candidates who write $6-\cos(\ldots$, no further marks are available unless the lack of brackets is corrected on the next line of working.
4. Do not penalise the inclusion of '$+c$' at $\bullet^2$ or $\bullet^3$.
5. Do not penalise the continued appearance of the integral sign or 'dx' after integrating.
6. Accept $\left(-2\cos\left(\frac{\pi}{2}\right)\right)-\left(-2\cos(0)\right)$ or $\left(-2\cos(3\times60-90)\right)-\left(-2\cos(3\times30-90)\right)$ for $\bullet^3$.
7. $\bullet^4$ is only available where candidates have considered both limits within a trigonometric function.
8. Where candidates use a mixture of degrees and radians, for example, $\left(-2\cos\left(3\times60°-\frac{\pi}{2}\right)\right)-\left(-2\cos\left(3\times30°-\frac{\pi}{2}\right)\right)$, $\bullet^3$ is not awarded. However, $\bullet^4$ is still available.

Commonly Observed Responses:

Candidate A - incomplete integration		Candidate B - integrating over two lines	
$-6\cos\left(3x-\frac{\pi}{2}\right)$	$\bullet^1$ ✓ $\bullet^2$ ^	$\int 6\sin\left(3x-\frac{\pi}{2}\right)dx$	
$\left(-6\cos\left(3\times\frac{\pi}{3}-\frac{\pi}{2}\right)\right)-\left(-6\cos\left(3\times\frac{\pi}{6}-\frac{\pi}{2}\right)\right)$	$\bullet^3$ ✓$_1$	$=-6\cos\left(3x-\frac{\pi}{2}\right)$	$\bullet^1$ ✓
6	$\bullet^4$ ✓$_1$	$=-6\cos\left(3x-\frac{\pi}{2}\right)\times\frac{1}{3}$	$\bullet^2$ ✗
		$\bullet^3$ and $\bullet^4$ are available	

Question	Generic scheme	Illustrative scheme	Max mark
10. (continued)			

Commonly Observed Responses (continued):

Candidate C - integrated in part $6\cos\left(3x-\frac{\pi}{2}\right)\times\frac{1}{3}$ •1 ✗ •2 $\checkmark_1$ •3 and •4 are available	**Candidate D - integrated in part** $-6\cos\left(3x-\frac{\pi}{2}\right)\times 3$ •1 ✓ •2 ✗ •3 and •4 are available
Candidate E - insufficient evidence of integration $6\cos\left(3x-\frac{\pi}{2}\right)$ •1 ✗ •2 ^ •3 and •4 are **not** available	**Candidate F - invalid approach** $\pm 6\sin\left(3x-\frac{\pi}{2}\right)\times\frac{1}{3}$ •1 ✗ •2 ✗ •3 and •4 are **not** available
Candidate G - working in degrees before integrating $\int_{30}^{60} 6\sin(3x-90)\,dx$ $-6\cos(3x-90)\times\frac{1}{3}$ •1 ✗ •2 ✗ $(-2\cos(90))-(-2\cos(0))$ •3 $\checkmark_1$ 2 •4 $\checkmark_1$	**Candidate H - BEWARE** $-6\cos\left(3x-\frac{\pi}{2}\right)\times\frac{1}{3}$ •1 ✓ •2 ✓ $\left(-2\cos\left(3\times\frac{\pi}{3}-\frac{\pi}{2}\right)\right)-\left(-2\cos\left(3\times\frac{\pi}{6}-\frac{\pi}{2}\right)\right)$ •3 ✓ $\left(-2\cos\left(\frac{\pi}{2}\right)\right)-(-2\cos\quad)$ •4 ✗ 2

Question			Generic scheme	Illustrative scheme	Max mark
11.	(a)	(i)	**Method 1** •¹ know to use $x=-2$ in synthetic division •² complete division, interpret result and state conclusion	**Method 1** •¹ $\begin{array}{r\|rrrr} -2 & 1 & 7 & 18 & 16 \\ & & & & \\ \hline & 1 & & & \end{array}$ •² $\begin{array}{r\|rrrr} -2 & 1 & 7 & 18 & 16 \\ & & -2 & -10 & -16 \\ \hline & 1 & 5 & 8 & 0 \end{array}$ Remainder $=0 \therefore (x+2)$ is a factor	2
			Method 2 •¹ know to substitute $x=-2$ •² complete evaluation, interpret result and state conclusion	**Method 2** •¹ $(-2)^3+7(-2)^2+18(-2)+16$ •² $=0 \therefore (x+2)$ is a factor	

Notes:

1. Communication at •² must be consistent with working at that stage i.e. candidate's working must arrive legitimately at 0 before •² can be awarded.
2. The conclusion for (a)(i) must be stated in (a)(i).
3. Accept any of the following for •²:
    - '$f(-2)=0$ so $(x+2)$ is a factor'
    - 'since remainder = 0, it is a factor'
    - the '0' from any method linked to the word 'factor' by 'so', 'hence', ∴, →, ⇒ etc
4. Do not accept any of the following for •²:
    - double underlining the '0' or boxing the '0' without comment
    - '$x=-2$ is a factor', '… is a root'
    - the word 'factor' only, with no link

Commonly Observed Responses:

Candidate A - grid method

	x^2	$5x$	8
x	x^3	$5x^2$	
2	$2x^2$		

•¹ ✓

	x^2	$5x$	8
x	x^3	$5x^2$	$8x$
2	$2x^2$	$10x$	16

with no remainder

$\therefore (x+2)$ is a factor •² ✓

Candidate B - grid method

	x^2	$5x$	8
x	x^3	$5x^2$	
2	$2x^2$		

•¹ ✓

	x^2	$5x$	8
x	x^3	$5x^2$	$8x$
2	$2x^2$	$10x$	16

$\therefore (x+2)(x^2+5x+8)=x^3+7x+18x+16$

$\therefore (x+2)$ is a factor •² ✓

Question			Generic scheme	Illustrative scheme	Max mark
11.	(a)	(ii)	•[3] use discriminant •[4] evaluate discriminant and give valid reason	•[3] $5^2 - 4(1)(8)$ •[4] $-7 < 0$, $\Rightarrow x^2 + 5x + 8$ does not factorise (so $(x+2)$ is the only linear factor)	**2**

Notes:

5. Evidence for •[3] may appear in the quadratic formula.
6. Accept $b^2 - 4ac = -7$ for •[3].
7. Do not penalise candidates who replace "$x^2 + 5x + 8$" with "the quadratic factor".
8. At •[4] accept interpretations such as "no real roots", "no (further) factors" and "cannot factorise (further)" with comparison with 0.
9. Where the quadratic factor used in •[3] can be factorised, •[4] is not available.

Commonly Observed Responses:

Candidate C - no reference to discriminant $-7 < 0$ •[3] ^ $\Rightarrow$ the quadratic factor does not factorise •[4] ^ $\Rightarrow (x+2)$ is the only linear factor	**Candidate D - minimal reference to discriminant** $-7 < 0$ •[3] ^ discriminant < 0 $\Rightarrow$ the quadratic factor does not factorise •[4] ✓ $\Rightarrow (x+2)$ is the only linear factor
Candidate E - no evaluation of discriminant $b^2 - 4ac < 0$ •[3] ^ •[4] ^ $\Rightarrow$ the quadratic factor does not factorise $\Rightarrow (x+2)$ is the only linear factor	**Candidate F - no evaluation of discriminant** Product = 8 and Sum = 5 No integer pair have this product and sum so the quadratic factor does not factorise •[3] ^ •[4] ^

Question			Generic scheme	Illustrative scheme	Max mark
11.	(b)		•[5] equate curves	•[5] $2x^3+20x^2+27x+9=6x^2-9x-23$	**4**
			•[6] rearrange and link to (a)	•[6] $2x^3+14x^2+36x+32=0$ leading to $2\left(x^3+7x^2+18x+16\right)=0$	
			•[7] find x-coordinate	•[7] $x=-2$	
			•[8] find y-coordinate	•[8] $y=19$	

Notes:

10. At •[6] do not penalise candidates who have divided the cubic equation by 2.
11. '$=0$' must appear in at least one of the equations at the •[6] stage.
12. •[7] may not be awarded for working in part (a).
13. Where an error has occurred at the •[5] or •[6] stage, •[7] and •[8] are still available for finding **all** the solutions (the discriminant need not be used for •[8] to be awarded on follow-through).
14. Where candidates "factorise" an irreducible quadratic or use another invalid strategy, •[7] and •[8] are not available.

Commonly Observed Responses:

Candidate G - not explicitly using part (a)

$2x^3+20x^2+27x+9=6x^2-9x-23$ •[5] ✓

$2x^3+14x^2+36x+32=0$

$$\begin{array}{r|rrrr} -2 & 2 & 14 & 36 & 32 \\ & & -4 & -20 & -32 \\ \hline & 2 & 10 & 16 & 0 \end{array}$$

$(x+2)\left(2x^2+10x+16\right)=0$ •[6] ✓

$2(x+2)\left(x^2+5x+8\right)=0$

To gain •[7], x^2+5x+8 must also be stated in (a) or the discriminant used.
Also accept $x=-2$ or $x+2=0$,
AND $x^2+5x+8=0$ for •[6]
(or without $=0$ if previously included)

$x=-2$ •[7] ✓
$y=19$ •[8] ✓

Candidate H - not using part (a)

$2x^3+20x^2+27x+9=6x^2-9x-23$ •[5] ✓

$2x^3+14x^2+36x+32=0$

$$\begin{array}{r|rrrr} -2 & 2 & 14 & 36 & 32 \\ & & -4 & -20 & -32 \\ \hline & 2 & 10 & 16 & 0 \end{array}$$

$(x+2)\left(2x^2+10x+16\right)$ •[6] ✓

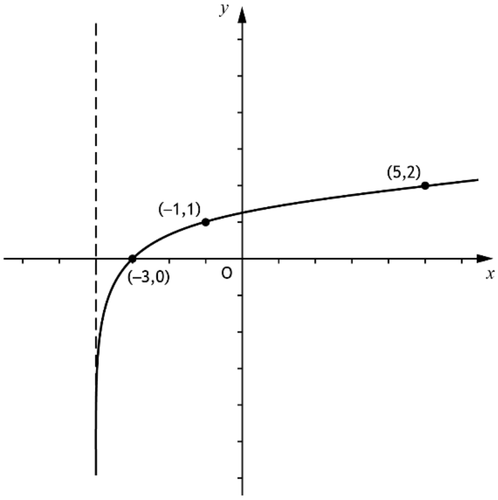
Also accept $x=-2$ or $x+2=0$,
AND $2x^2+10x+16=0$ for •[6]
(or without $=0$ if previously included)

$x=-2$ and $10^2-4(2)(16)=-28<0$ so
$x=-2$ is the only root •[7] ✓
$y=19$ •[8] ✓

Question		Generic scheme	Illustrative scheme	Max mark
11.	(b)	(continued)		

Commonly Observed Responses (continued):

Candidate I - no evidence of link to part (a)	Candidate J - subtracting equations
$2x^3+20x^2+27x+9=6x^2-9x-23$ •5 ✓ $2x^3+14x^2+36x+32=0$ •6 ^ •7 ^ $x=-2$ $y=19$ •8 ✓1	$0=(2x^3+20x^2+27x+9)-(6x^2-9x-23)$ •5 ✓ $0=2x^3+14x^2+36x+32$ $0=2(x^3+7x^2+18x+16)$ •6 ✓ However… $y=(2x^3+20x^2+27x+9)-(6x^2-9x-23)$ •5 ✗ $0=2(x^3+7x^2+18x+16)$ •6 ✓1

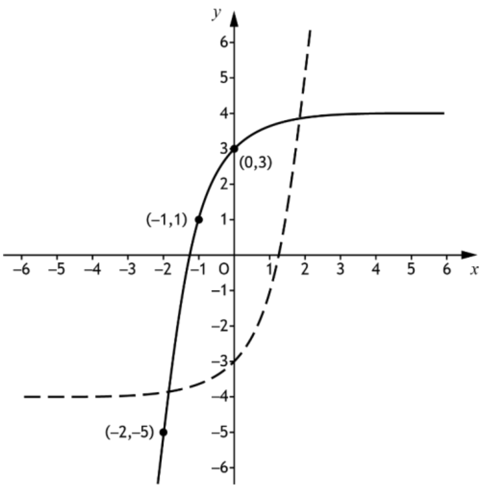
Question			Generic scheme	Illustrative scheme	Max mark
12.	(a)		•¹ graph reflected in $y = x$ i.e. generally increasing for $-3 \le x \le 5$ and passing through (−3,0), (−1,1) and (5,2) •² increasing for $x < -3$ (and for $x > 5$) and approaching $x = -4$ from the right	•¹, •²  	**2**

Notes:

1. •¹ is not available where the graph has a maximum turning point where $-1 < x < 5$ and •² is not available where the graph has a maximum turning point where $5 \le x \le 6$.
2. •² is only available where •¹ has been awarded.
3. The line $x = -4$ does not need to be shown.
4. For a rotation, award 0/2 – for example see Candidate A.
5. Do not penalise graphs which do not extend beyond (5,2).

Commonly Observed Responses:

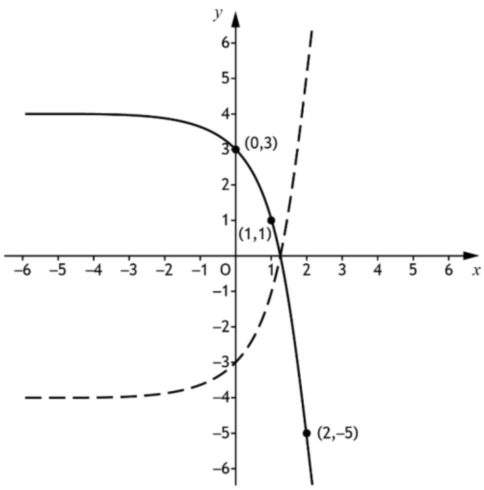
Candidate A - rotation about O



•¹ × •² ×

No further marks in this question can be inferred *from this graph*.
In part (b), candidates may work with the original graph and gain full marks for $b = 4$ and $a = 3$ (with or without working) and $x > -4$.

Candidate B - reflection in x-axis



•¹ × •² ×

No further marks in this question can be inferred *from this graph*.
In part (b), candidates may work with the original graph and gain full marks for $b = 4$ and $a = 3$ (with or without working) and $x > -4$.

Question			Generic scheme	Illustrative scheme	Max mark
12.	(b)	(i)	$\bullet^3$ determine value of b $\bullet^4$ determine value of a	$\bullet^3$ $b = 4$ $\bullet^4$ $a = 3$	**2**
		(ii)	$\bullet^5$ state domain	$\bullet^5$ $x > -4, \quad (x \in \mathbb{R})$	**1**

Notes:

6. a and b may be calculated independently of each other.
7. The values of a or b can be calculated using $a = b - 1$ where $b > 1$.
8. Accept '$f(x) > -4$' for $\bullet^5$.
9. For $\bullet^5$, the domain can be calculated using $x > -b$.
10. For $\bullet^5$, do not penalise the omission of $x \in \mathbb{R}$.
11. For $\bullet^5$, the domain can be from the original graph, the candidate's transformed graph or from the value of b in (b)(i). However, refer back to Candidates A and B.

Commonly Observed Responses:

Candidate C - Values of *a* and *b* in equation		**Candidate D - Incorrect values of *a* and *b***	
$\log_3(x+4)$	$\bullet^3$ ✓ $\bullet^4$ ✓	$\log_3(x+4)$	$\bullet^3$ ✗
		$b = 3$	$\bullet^4$ ✗
		$x > -3$	$\bullet^5$ ✓$_1$

[END OF MARKING INSTRUCTIONS]