National
Qualifications
2026

2026 Mathematics

Advanced Higher Paper 1

Question Paper Finalised Marking Instructions

These marking instructions have been prepared by examination teams for use by Qualifications Scotland appointed markers when marking external course assessments.

General marking principles for Advanced Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example $6\square 6=12$, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.

This is no longer a solution of a quadratic equation, so the mark is not awarded.

$$x^2+5x+7=9x+4$$
$$x-4x+3=0$$
$$x=1$$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$x^2+5x+7=9x+4$$
$$x-4x+3=0$$
$$(x-3)(x-1)=0$$
$$x=1 \text{ or } 3$$

(i) Horizontal/vertical marking
If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x = 2$	$x = -4$
$\bullet^6$	$y = 5$	$y = -7$

Horizontal: $\bullet^5$ $x = 2$ and $x = -4$
$\bullet^6$ $y = 5$ and $y = -7$

Vertical: $\bullet^5$ $x = 2$ and $y = 5$
$\bullet^6$ $x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$. or $1\frac{1}{4}$

$\frac{43}{1}$ must be simplified to 43

$\frac{15}{0.3}$ must be simplified to 50

$\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 144 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example
  $\left(x^3+2x^2+3x+2\right)\left(2x+1\right)$ written as
  $\left(x^3+2x^2+3x+2\right)\times 2x+1$
  $=2x^4+5x^3+8x^2+7x+2$ gains full credit
- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Marking Instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.	(a)		•¹ evidence of use of the product rule and one term correct [1]	•¹ $12x^3\sec 2x+\ldots$ **or** $\ldots+3x^4\times\sec 2x\tan 2x\times 2$	**2**
			•² complete differentiation [1]	•² $12x^3\sec 2x+6x^4\sec 2x\tan 2x$	

Notes:

1. For a candidate who produces only one term, award 0/2.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•³ evidence of use of the quotient rule with denominator and one term of numerator correct [1]	•³ $\dfrac{5e^{5x}(2x+1)-\ldots}{(2x+1)^2}$ **or** $\dfrac{\ldots-2e^{5x}}{(2x+1)^2}$	**3**
			•⁴ complete differentiation	•⁴ $\dfrac{5e^{5x}(2x+1)-2e^{5x}}{(2x+1)^2}$	
			•⁵ simplify [2]	•⁵ $\dfrac{10xe^{5x}+3e^{5x}}{(2x+1)^2}$	

Notes:

1. For a candidate who equates $f'(x)$ to $f(x)$, •³ is not available.
2. Some simplification of the numerator involving the collection of like terms is required for the award of •⁵.

Commonly Observed Responses:

Product Rule:

•³ $5e^{5x}(2x+1)^{-1}+\ldots$

•⁴ $\ldots+e^{5x}(-1)(2x+1)^{-2}$

•⁵ $5e^{5x}(2x+1)^{-1}-2e^{5x}(2x+1)^{-2}$

Question			Generic scheme	Illustrative scheme	Max mark
2.			•¹ set up augmented matrix [1]	•¹ $\left(\begin{array}{ccc\|c} 1 & 1 & -1 & 9 \\ 2 & -1 & 3 & -2 \\ 3 & 2 & -2 & 21 \end{array}\right)$	4
			•² obtain two zeros [1,2]	•² eg $\left(\begin{array}{ccc\|c} 1 & 1 & -1 & 9 \\ 0 & -3 & 5 & -20 \\ 0 & -1 & 1 & -6 \end{array}\right)$	
			•³ complete row operations [1]	•³ eg $\left(\begin{array}{ccc\|c} 1 & 1 & -1 & 9 \\ 0 & -3 & 5 & -20 \\ 0 & 0 & -2 & 2 \end{array}\right)$	
			•⁴ find values of x, y and z [3,4]	•⁴ ʷ $x = 3,\ y = 5,\ z = -1$	

Notes:

1. Where a candidate equates a 3×3 matrix to a 3×1 matrix, •¹ is not available. Otherwise, accept eg $x, y, z, =$ left in.
2. Only Gaussian elimination (ie a systematic approach using EROs) is acceptable for the award of •² and •³.
3. Where a candidate obtains a final matrix which exhibits inconsistency, •⁴ is not available.
4. Where a candidate obtains a final matrix which exhibits redundancy, the equation of a line must be produced for the award of •⁴.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
3.	(a)		$\bullet^1$ calculate modulus or argument [1,2]	$\bullet^1$ 2 **or** $\frac{\pi}{6}$	**2**
			$\bullet^2$ write in polar form [1,2]	$\bullet^2$ $2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$	

Notes:

1. Accept arguments expressed in degrees provided the degree symbol appears at least once in part (a) or (b). Otherwise, withhold $\bullet^2$.
2. Any working leading to the calculation of the argument or modulus must be consistent (General Marking Principle (n)). For example, do not accept $\tan\frac{1}{\sqrt{3}}=\frac{\pi}{6}$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		$\bullet^3$ apply de Moivre's theorem [1,2,3]	$\bullet^3$ $2^3\left(\cos\frac{3\pi}{6}+i\sin\frac{3\pi}{6}\right)$	**2**
			$\bullet^4$ complete process [1,4]	$\bullet^4$ $8i$	

Notes:

1. Where a candidate does not use de Moivre's theorem, award 0/2.
2. For the award of $\bullet^3$, there must be a single numerical argument.
3. Where, in part (a), a candidate produces a modulus of ± 1, they must explicitly demonstrate that it has been cubed for the award of $\bullet^3$.
4. Where a candidate obtains a number which is not purely imaginary, $\bullet^4$ is not available.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
4.			•1 construct auxiliary equation 1 •2 find general solution 2 •3 differentiate general solution 3 •4 form simultaneous equations •5 state particular solution 4	•1 $2m^2-3m+1=0$ •2 $y=Ae^{\frac{1}{2}x}+Be^x$ •3 $\frac{dy}{dx}=\frac{1}{2}Ae^{\frac{1}{2}x}+Be^x$ •4 $2=A+B$ $-1=\frac{1}{2}A+B$ •5 $y=6e^{\frac{1}{2}x}-4e^x$	5

Notes:

1. Where "$=0$" does not appear in the auxiliary equation, •1 is not available.
2. Disregard the omission of "$y=$" at •2.
3. Where a candidate does not give an expression for the derivative, •3 may be awarded for $-1=\frac{1}{2}A+B$.
4. Do not award •5 if "$y=$" does not appear at that stage.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
5.	(a)		$\bullet^1$ find expression for determinant [1]	$\bullet^1$ $3x+10$	**1**

Notes:

1. Disregard poor notation (eg equating a matrix to its determinant), but do not accept $3x+10=0$.

Commonly Observed Responses:

	(b)		$\bullet^2$ state determinant of B [1,2]	$\bullet^2$ 4	**1**

Notes:

1. Disregard poor notation.
2. Where a candidate has produced an incorrect expression in (a), $\bullet^2$ is available only if simplification is required (see COR).

Commonly Observed Responses:

$\frac{12x+40}{3x+6} = \frac{4(3x+10)}{3(x+2)}$ Do not award $\bullet^2$.

Question			Generic scheme	Illustrative scheme	Max mark
5.	(c)		**Method 1**	**Method 1**	2
			•3 rewrite B^{-1} 1,2,3	•3 $\frac{1}{4}\begin{pmatrix} 4 & -4 \\ -5 & 6 \end{pmatrix}$	
			•4 state B 1,2,3	•4 $B=\begin{pmatrix} 6 & 4 \\ 5 & 4 \end{pmatrix}$	
			Method 2	**Method 2**	
			•3 state expression for B^{-1} 1,2,4	•3 $\frac{1}{4}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$	
			•4 state B 1,2	•4 $B=\begin{pmatrix} 6 & 4 \\ 5 & 4 \end{pmatrix}$	
			Method 3	**Method 3**	
			•3 begin to invert B^{-1} 1,2	•3 $\begin{pmatrix} \frac{3}{2} & 1 \\ \frac{5}{4} & 1 \end{pmatrix}$	
			•4 state B 1,2,5	•4 $B=\begin{pmatrix} 6 & 4 \\ 5 & 4 \end{pmatrix}$	

Notes:

1. Where a candidate attempts to use division by a matrix, no further marks are available.
2. For the correct answer without working, award 2/2 if the candidate confirms their answer by evaluating $B^{-1}B$ or BB^{-1} to obtain I . Otherwise, award 1/2.
3. In method 1, the appearance of $\begin{pmatrix} 4 & -4 \\ -5 & 6 \end{pmatrix}$ on its own is not sufficient for the award of •3. It is, however, sufficient justification for the award of •3 and •4 provided it is followed by the correct matrix for B .
4. In method 2, where a candidate has obtained an incorrect answer in part (b), •3 may be awarded if the scalar multiplier is the reciprocal of that, or is the determinant of the given matrix $\left(\frac{1}{4}\right)$.
5. In method 3, the award of •4 requires a matrix at •3 which contains all of $(\pm)1,(\pm)1,(\pm)\frac{3}{2},(\pm)\frac{5}{4}$.
   Each element must be individually multiplied by the determinant of B to produce a matrix at •4.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
6.	(a)		•1 differentiate •2 rewrite integral and expand brackets [1] •3 integrate [1,2]	•1 $\frac{du}{dx}=1$ **or** $du=dx$ •2 $\int u^5+u^4du$ •3 $\frac{1}{6}(x-1)^6+\frac{1}{5}(x-1)^5+c$	3

Notes:

1. Other than in COR C below, where a candidate attempts to integrate an expression containing x and u, •2 and •3 are not available.
2. Where the integration involves a single power, •3 is not available.
3. Disregard the omission of the constant of integration.

Commonly Observed Responses:

A: Integration by parts with no substitution

•1 Do not award

•2 $\frac{1}{5}x(x-1)^5-\int\frac{1}{5}(x-1)^5\,dx$

•3 $\frac{1}{5}x(x-1)^5-\frac{1}{30}(x-1)^6+c$

B: Integration by parts following substitution

•2 $\frac{1}{5}u^5(u+1)-\dots$

•3 $\frac{1}{5}x(x-1)^5-\frac{1}{30}(x-1)^6+c$

C: Integration by parts following substitution (with x as a function of u, and since $\frac{du}{dx}=1$)

•2 $\frac{1}{5}u^5x-\int\frac{1}{5}u^5\,du$

•3 $\frac{1}{5}x(x-1)^5-\frac{1}{30}(x-1)^6+c$

Question			Generic scheme	Illustrative scheme	Max mark
6.	(b)		•[4] correct form of integral, including limits [1]	•[4] $\pi\int_0^1 y^2\,dx$	4
			•[5] substitute for y^2 and simplify [2]	•[5] $\pi\int_0^1 4x\left(x-1\right)^4 dx$	
			•[6] integrate using result from (a) and include limits [2,3]	•[6] $4\pi\left[\frac{1}{6}\left(x-1\right)^6+\frac{1}{5}\left(x-1\right)^5\right]_0^1$	
			•[7] evaluate [2,3,4,5]	•[7] $\frac{2\pi}{15}$	

Notes:

1. For the award of •[4], limits and dx must appear at some point.
2. Where a candidate writes dy instead of dx but integrates with respect to x, •[5], •[6] and •[7] are still available.
3. Where a candidate produces an expression with more than one variable, •[6] and •[7] are unavailable. Do not treat this as a repeated error from (a).
4. Other than in COR A, B or C in (a), for the award of •[7], the evaluation must involve the addition or subtraction of two fractions with different denominators.
5. Where a candidate produces a final answer which is not positive, •[7] is not available.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)		$\bullet^1$ state second root	$\bullet^1$ $2-i$	1

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		$\bullet^2$ obtain two linear factors	$\bullet^2$ $z-(2+i),\ z-(2-i)$	5
			$\bullet^3$ obtain quadratic factor	$\bullet^3$ z^2-4z+5	
			$\bullet^4$ set up and begin algebraic division or equivalent [1]	$\bullet^4$ $\begin{array}{r} z^2 \\ z^2-4z+5\overline{\big)\,z^4-2z^3-z^2+2z+10} \\ z^4-4z^3+5z^2 \end{array}$	
			$\bullet^5$ obtain second quadratic factor [1,2]	$\bullet^5$ z^2+2z+2	
			$\bullet^6$ obtain remaining two roots [2]	$\bullet^6$ $-1\pm i$	

Notes:

1. Where, having correctly stated the first quadratic factor, a candidate produces the second with no intermediate working, award $\bullet^4$ and $\bullet^5$.
2. Where a candidate produces a second quadratic expression which is not a factor, $\bullet^5$ is not available but $\bullet^6$ may still be, provided the expression has a negative discriminant.

Question		Generic scheme	Illustrative scheme	Max mark
7.	(b)	(continued)		

Commonly Observed Responses:

Synthetic division

Division by $2-i$:

$\bullet^2$ $$2-i \,\big|\; 1 \quad -2 \quad -1 \quad 2 \quad 10$$

$\bullet^3$
$$\begin{array}{r|rrrr|r}
2-i & 1 & -2 & -1 & 2 & 10 \\
 & & 2-i & -1-2i & -6-2i & -10 \\
\hline
 & 1 & -i & -2-2i & -4-2i & 0
\end{array}$$

$\bullet^4$
$$\begin{array}{r|rrr|r}
2+i & 1 & -i & -2-2i & -4-2i \\
 & & 2+i & 4+2i & 4+2i \\
\hline
 & 1 & 2 & 2 & 0
\end{array}$$

Division by $2+i$:

$\bullet^2$ $$2+i \,\big|\; 1 \quad -2 \quad -1 \quad 2 \quad 10$$

$\bullet^3$
$$\begin{array}{r|rrrr|r}
2+i & 1 & -2 & -1 & 2 & 10 \\
 & & 2+i & -1+2i & -6+2i & -10 \\
\hline
 & 1 & i & -2+2i & -4+2i & 0
\end{array}$$

$\bullet^4$
$$\begin{array}{r|rrr|r}
2-i & 1 & i & -2+2i & -4+2i \\
 & & 2-i & 4-2i & 4-2i \\
\hline
 & 1 & 2 & 2 & 0
\end{array}$$

Grid Method

$\bullet^4$
$$\begin{array}{r|rrr}
 & z^2 & -4z & +5 \\
\hline
z^2 & z^4 & -4z^3 & +5z^2 \\
 & & & \\
+2 & & & 10
\end{array}$$

[END OF MARKING INSTRUCTIONS]