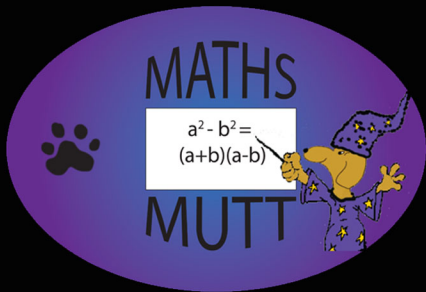


# SQA Revision



## Advanced Higher Maths Checklist

## Advanced Higher Maths Checklist

## Apps 1.1 Applying algebraic skills to the binomial theorem and to complex numbers.

Factorials	<input type="checkbox"/>
Permutations	<input type="checkbox"/>
Combinations	<input type="checkbox"/>
Pascal's Triangle.	<input type="checkbox"/>
Binomial Co-efficients	<input type="checkbox"/>
Binomial Co-efficient equations	<input type="checkbox"/>
Binomial Expansions	<input type="checkbox"/>
Binomial Theorem	<input type="checkbox"/>
Probability and the Binomial Theorem	<input type="checkbox"/>
Binomial Expansions and e	<input type="checkbox"/>
Complex numbers	<input type="checkbox"/>
Complex conjugate	<input type="checkbox"/>
Arithmetic operations	<input type="checkbox"/>
Argand diagrams	<input type="checkbox"/>
Loci	<input type="checkbox"/>
Polar form	<input type="checkbox"/>
De Moivre's theorem	<input type="checkbox"/>
Roots of a complex number	<input type="checkbox"/>

$$\begin{aligned}
 (x+y)^5 &= 1x^5y^0 + 5x^4y^1 + 10x^3y^2 + 10x^2y^3 + 5x^1y^4 + 1x^0y^5 \\
 &= {}^5C_0x^5y^0 + {}^5C_1x^4y^1 + {}^5C_2x^3y^2 + {}^5C_3x^2y^3 + {}^5C_4x^1y^4 + {}^5C_5x^0y^5 \\
 &= \binom{5}{0}x^5y^0 + \binom{5}{1}x^4y^1 + \binom{5}{2}x^3y^2 + \binom{5}{3}x^2y^3 + \binom{5}{4}x^1y^4 + \binom{5}{5}x^0y^5
 \end{aligned}$$

## Apps 1.2 Applying algebraic skills to sequences and series.

Recurrence relations	<input type="checkbox"/>
Fixed points	<input type="checkbox"/>
Arithmetic sequences	<input type="checkbox"/>
Arithmetic series	<input type="checkbox"/>
nth term of arithmetic sequence	<input type="checkbox"/>
Sum to n terms of an arithmetic sequence	<input type="checkbox"/>
Geometric sequences	<input type="checkbox"/>
Geometric series	<input type="checkbox"/>
Sum to n terms of a geometric sequence	<input type="checkbox"/>
Infinite series	<input type="checkbox"/>
Sum to infinity - geometric series	<input type="checkbox"/>
Sum first n natural numbers	<input type="checkbox"/>
Sigma notation - Rules	<input type="checkbox"/>
Common series - sigma notation	<input type="checkbox"/>
Power series	<input type="checkbox"/>
D'alembert's ratio test	<input type="checkbox"/>
Absolute convergence	<input type="checkbox"/>
Fibonacci series	<input type="checkbox"/>
Centre of convergence	<input type="checkbox"/>
Taylor's series	<input type="checkbox"/>
Maclaurin's Series	<input type="checkbox"/>
Maclaurin expansion	<input type="checkbox"/>

$$u_n = a + (n-1)d$$

$$S_n = \frac{1}{2}n(2a + (n-1)d)$$

$$u_n = ar^{(n-1)}$$

$$S_n = \frac{a(1-r^n)}{1-r}, \quad r \neq 1$$

## Advanced Higher Maths Checklist

## Apps 1.3 Applying algebraic skills to summation and mathematical proof.

Standard series	<input type="checkbox"/>	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
Examples	<input type="checkbox"/>	
Iteration	<input type="checkbox"/>	
Iteration - graphical method	<input type="checkbox"/>	
First order process	<input type="checkbox"/>	$e^x = \sum_{n=0}^{\infty} \frac{(ax)^n}{n!} = \sum_{n=0}^{\infty} a^n \frac{x^n}{n!} = 1 + \frac{(ax)}{1!} + \frac{(ax)^2}{2!} + \frac{(ax)^3}{3!} + \frac{(ax)^4}{4!} + \dots$
Second order process	<input type="checkbox"/>	
Rule of false position	<input type="checkbox"/>	
Newton-Raphson iteration	<input type="checkbox"/>	
Proof by induction	<input type="checkbox"/>	

## Apps 1.4 Applying algebraic and calculus skills to properties of functions.

Modulus function	<input type="checkbox"/>	Odd functions have half-turn symmetry about the origin, so $f(-x) = -f(x)$
Inverse functions	<input type="checkbox"/>	
Polynomials	<input type="checkbox"/>	
Extrema	<input type="checkbox"/>	
Concavity and points of inflexion	<input type="checkbox"/>	Even functions are symmetrical about the y – axis so $f(-x) = f(x)$
Odd and Even functions	<input type="checkbox"/>	
Asymptotes	<input type="checkbox"/>	

## Apps 1.5 Applying algebraic and calculus skills to problems.

Rectilinear motion	<input type="checkbox"/>	
Distance, velocity & acceleration	<input type="checkbox"/>	Since
Applications of differential equations	<input type="checkbox"/>	$a = \frac{dv}{dt} \quad , \quad v = \int a dt$
Extrema of functions	<input type="checkbox"/>	
Approximating roots	<input type="checkbox"/>	$v = \frac{ds}{dt} \quad , \quad s = \int v dt$
Uses of parametric equations	<input type="checkbox"/>	
Equations of motion	<input type="checkbox"/>	
	<input type="checkbox"/>	

## Notes

## Advanced Higher Maths Checklist

## GPS 1.1 Applying algebraic skills to matrices and systems of equations.

- Augmented matrix
- Elementary Row Operations
- Gaussian elimination
- Matrix equality
- Matrix addition and subtraction
- Matrix notation
- Matrix elements
- Matrix Scalar multiplication
- Transpose of a matrix
- Matrix multiplication
- Determinant :2 x 2 matrix
- Determinant: 3 x 3 matrix
- Cofactor of a 3 x 3 matrix
- Adjoint of a 3 x 3 matrix
- Inverse of a square matrix
- Product of a square matrix and its inverse
- Using Gaussian elimination to find inverse of matrix
- Diagonal matrix
- Null matrix
- Unit matrix
- Reflections
- Rotations
- Scalings

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$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

has determinant

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

$$\det(A) = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$= a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

## Notes

## Advanced Higher Maths Checklist

## GPS 1.2 Applying algebraic and geometric skills to vectors.

- ☐ Direction Ratios and Cosines
- ☐ Vector product
- ☐ Right Hand Screw Rule
- ☐ Vector Product properties
- ☐ Components
- ☐ Scalar triple product
- ☐ Intersection of two lines
- ☐ Intersection of two planes
- ☐ The distance from a point to a plane
- ☐ The distance from a point to a line
- ☐ Equations of a line
- ☐ Planes
- ☐ Planes in space
- ☐ Vector equations
- ☐ The angle between two planes
- ☐ The distance between parallel planes
- ☐ Coplanar vectors

$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}| |\mathbf{b}| \sin \theta$$

describes the area of the parallelogram defined by  $\mathbf{a}$  and  $\mathbf{b}$

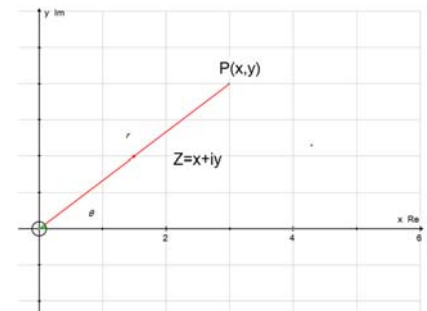
$\mathbf{a} \times \mathbf{b}$  is the determinant of the matrix  $\begin{pmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{pmatrix}$

$$\therefore \mathbf{a} \times \mathbf{b} = \begin{pmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{pmatrix}$$

$$= (a_2 b_3 - a_3 b_2)\mathbf{i} - (a_1 b_3 - a_3 b_1)\mathbf{j} + (a_1 b_2 - a_2 b_1)\mathbf{k}$$

## GPS 1.3 Applying geometric skills to complex numbers.

- ☐ Complex numbers
- ☐ Complex conjugate
- ☐ Arithmetic operations
- ☐ Argand diagrams
- ☐ Loci
- ☐ Polar form
- ☐ De Moivre's theorem
- ☐ Roots of a complex number



## GPS 1.4 Applying algebraic skills to number theory.

- ☐ The division algorithm
- ☐ The euclidean algorithm
- ☐ Diophantine equations
- ☐ Pythagorean Triples
- ☐ Number bases
- ☐ The division algorithm

$$\frac{a}{b} = q + \frac{r}{b}$$

$a$  divided by  $b$  gives a quotient and remainder.  
The quotient  $q$  and remainder  $r$  are integers

$$0 \leq r < |b|$$

## GPS 1.5 Applying algebraic and geometric skills to methods of proof.

- ☐ Mathematical proof
- ☐ Direct proof
- ☐ Proof by contradiction
- ☐ Proof by contrapositive
- ☐ Proof by induction

$$b|a \quad b \text{ divides } a$$

$$\text{so } a = kb, \quad k \in \mathbb{W}$$

## Advanced Higher Maths Checklist

## MAC 1.1 Applying algebraic skills to partial fractions.

Rational fractions

Partial fractions

Distinct linear factors

Repeated linear factors

Irreducible quadratic factor

$$\begin{aligned} \frac{3x^2 - 11x + 5}{(x-2)(x-1)^2} &= \frac{A}{x-2} + \frac{B}{x-1} + \frac{C}{(x-1)^2} \\ &= \frac{A(x-1)^2 + B(x-2)(x-1) + C(x-2)}{(x-2)(x-1)^2} \end{aligned}$$

## MAC 1.2 Applying calculus skills through techniques of differentiation.

Sec, cosec, cot

Higher Derivatives

Chain rule.

Product rule.

Quotient rule

Exponentials &amp; Logs

Differentiating inverse functions

Differentiating inverse trig functions

Inverse trig functions:Recap

Implicit &amp; explicit functions

Logarithmic functions

Parametric equations

Parametric equations:circles

Parametric constraint equations

Differentiating parametric

Second Derivative: parametric equations

Uses of parametric equations

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dt} \times \frac{dt}{dx}$$

If  $f(x) = g(x) \cdot h(x)$   
then  $f'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x)$

If  $f(x) = \frac{g(x)}{h(x)}$   
then  $f'(x) = \left( \frac{g(x)}{h(x)} \right)' = \frac{g'(x)h(x) - g(x)h'(x)}{(h(x))^2}$

$$f'x = \frac{y'(t)}{x'(t)}$$

$$f''x = \frac{x'(t)y''(t) - x''(t)y'(t)}{(x'(t))^3}$$

## Notes

## Advanced Higher Maths Checklist

### MAC 1.3 Applying calculus skills through techniques of integration.

Standard integrals	<input type="checkbox"/>	$\lim_{\delta x \rightarrow 0} \sum_{x=a}^b \pi y^2 dx = \int_a^b \pi y^2 dx$
$e^x$ and $1/x$	<input type="checkbox"/>	
Infinite integrals (discontinuities)	<input type="checkbox"/>	
$\sec^2 x$	<input type="checkbox"/>	$\int \frac{f'(x)}{f(x)} dx = \ln f(x)  + C$
Integrating Inverse trig functions	<input type="checkbox"/>	
Integration of rational functions	<input type="checkbox"/>	
Volumes of revolution	<input type="checkbox"/>	$\int f'(x)f(x)dx = \frac{1}{2}(f(x))^2 + C$
Integration by substitution.	<input type="checkbox"/>	
Integration by parts	<input type="checkbox"/>	

### MAC 1.4 Applying calculus skills to solving differential equations.

Differential equations	<input type="checkbox"/>	$\frac{dy}{dx} = f(x)g(y)$
First-order differential equations	<input type="checkbox"/>	$\Rightarrow \frac{dy}{g(y)} = f(x)dx$
Second-order homogenous differential equations	<input type="checkbox"/>	$\Rightarrow \int \frac{dy}{g(y)} = \int f(x)dx$
Second order non – homogeneous Differential Equations	<input type="checkbox"/>	

## Notes